A side-coupled photonic crystal filter with sidelobe suppression

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ABSTRACT This paper presents the theoretical model and the optimization method to suppress the sidelobes of side-coupled photonic crystal filters. Numerical verification shows a good agreement between the theoretical model and the finite-difference time-domain simulation, but the theoretic method does not involve the time-consuming computation. The theoretical model also presents a better physical image for choosing the critical parameters, such as the quality factor, phase shift and the number of resonators. Based on the theoretical model, two optimization methods (chirp and cascading) are proposed to deeply suppress the sidelobes. They also show more flexibility in controlling the bandwidth and steepness of the roll-off in the filter.

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1 Introduction

The study of photonic crystals (PhC) has attracted many researchers due to its potential application in optical integrated circuits and all-optical communication networks. Many devices based on PhC have been designed, such as the filters, the lasers and the optical switches [1–3]. Among these PhC devices, side-coupled resonators have been studied both theoretically and experimentally [4–9]. In order to obtain broad flat-top, many resonators should be cascaded, but the sidelobes become severe due to the finite uniform structures [7, 8].

Many techniques are proposed to suppress the sidelobes, and the basic idea is to break the intrinsic periodicity of the optical system. The first technique is the well-known apodization technique, which is widely used to design the grating filters by tapering the coupling coefficients [13]. Application of the apodization technique in PhC has been discussed in [9, 10]. Regarding the current PhC filter, the periodicity can be broken by modulating the coupling coefficients between resonators and the waveguide according to a specific function along the waveguide, such as the Gaussian function, the raised cosine function or the Hamming function. However, this method has to change the refractive index of the resonators continuously, which is difficult to fabricate. Making matters worse, the structure has to be very long to obtain an acceptable performance, counteracting the advantage of PhC on the compactness.

The second technique is the chirp technique, which is similar to the one widely used in fiber Bragg gratings [15]. It modulates the periodicity along the waveguide. The third technique is the cascading technique, which has been used in optical filter system [12]. It cascades many resonators, each with the different resonance frequency. The chirp technique and the cascading technique have been proven very effective to suppress the sidelobes of the conventional filters, but have not yet been applied to the PhC filters. On the other hand, sidelobe suppression has not been systematically studied even though many PhC filters have been investigated using the coupled mode theory (CMT) and the transfer matrix method (TMM).

Based on the above discussion, this paper aims at developing a theoretical method to reveal the critical parameters for the sidelobe suppression, and to find their relationship. In addition, the chirp technique and the cascading technique have been applied to design the PhC filters. This paper is organized as follows: the theoretical model based on the CMT and the TMM is set up in Sect. 2. 3 verifies the validity of the theoretical model by using the FDTD simulation, along with the demonstration of the superior filter performance given by the two optimization techniques, and 4 summaries all results.

2 Theoretical model

A general photonic crystal filter with the side-coupled resonator is shown in Fig. 1. It consists of $N$ resonators and a waveguide. The resonance frequency and the phase shift of every resonator are usually different from each other. In the following, we will adopt the CMT to establish the theoretical model for this kind of side-coupled filter.

The basic structure of the proposed reflection filter is shown in Fig. 2. The filter consists of two identical single mode resonators which are side-coupled to the PhC waveguide. Direct coupling between two neighboring resonators through evanescent wave tunneling is prohibited in this configuration. Therefore, the resonators can only interact on each other through the guided mode of the PhC waveguide. Based
on the CMT, the relation between one resonator and the waveguide can be expressed as \[ [S'_{+1}] [S'_{-1}] = T_2 \left[ \begin{array}{cc} e^{j\Phi} & 0 \\ 0 & e^{-j\Phi} \end{array} \right] T_1 \left[ \begin{array}{c} S_{+1} \\ S_{-1} \end{array} \right], \] (2)

where \( S_+ \), \( S_- \) represent the light propagating into and out of the resonator, respectively. \( \omega_0 \) and \( \gamma \) are the resonant frequency and the width of the resonator, and have the relationship \( \gamma = \omega_0 / Q \), where \( Q \) is the quality factor. If two resonators are coupled to the same waveguide as shown in Fig. 1, according to the CMT and the TMM, we could get

\[
\begin{bmatrix}
S_{+2} \\
S_{-2}
\end{bmatrix} = T_2 \begin{bmatrix} e^{j\Phi} & 0 \\ 0 & e^{-j\Phi} \end{bmatrix} T_1 \begin{bmatrix} S_{+1} \\
S_{-1}
\end{bmatrix},
\]

(3)

where phase shift \( \Phi = \beta L \), \( \beta \) is the propagation constant of the waveguide, and \( L \) is the distance between two reference planes as shown in Fig. 2. When \( N + 1 \) resonators are cascaded along the waveguide, the relationship between the resonators and the waveguide can be expressed as

\[
\begin{bmatrix}
S_{+/(N+1)} \\
S_{-/(N+1)}
\end{bmatrix} = T_{N+1} D_N T_N \cdots D_1 T_1 \begin{bmatrix} S_{+1} \\
S_{-1}
\end{bmatrix},
\]

(3)

where

\[
T_i = \begin{bmatrix}
1 - \frac{j\gamma}{\omega - \omega_0} & -\frac{j\gamma}{\omega - \omega_0} \\
\frac{j\gamma}{\omega - \omega_0} & 1 + \frac{j\gamma}{\omega - \omega_0}
\end{bmatrix}, \quad D_i = \begin{bmatrix} e^{j\Phi_i} & 0 \\
0 & e^{-j\Phi_i} \end{bmatrix}
\]

and \( i \) means the \( i \)th resonator, \( i = 1, 2, 3, ..., N + 1 \). If the resonators are identical, they have equal \( T \) and \( D \) matrix. Therefore,

\[
\begin{bmatrix}
S_{+(N+1)} \\
S_{-(N+1)}
\end{bmatrix} = T(DT)^N \begin{bmatrix} S_{+1} \\
S_{-1}
\end{bmatrix}.
\]

(4a)

Define:

\[
m \equiv DT = \begin{bmatrix} m_{11} & m_{12} \\
m_{21} & m_{22} \end{bmatrix},
\]

(4b)

\[
M \equiv T(DT)^N = \begin{bmatrix} M_{11} & M_{12} \\
M_{21} & M_{22} \end{bmatrix}.
\]

(4c)

Note that the matrix \( m \) is unimodular, i.e., \( m_{11}m_{22} - m_{12}m_{21} = 1 \). For \( N + 1 \) identical resonators, this relationship may be iterated by \( N \) times. Based on the Sylvester’s theorem [11], we could get

\[
\begin{bmatrix} m_{11} & m_{12} \\
\frac{m_{21}}{m_{12}} & \frac{m_{22}}{m_{12}} \end{bmatrix}^N = \frac{1}{\sin \theta} \begin{bmatrix} m_{11} \sin(N\theta) & m_{12} \sin(N\theta) \\
-m_{21} \sin(N\theta) & m_{22} \sin(N\theta) \end{bmatrix}.
\]

(5)

where \( \cos \theta = \frac{1}{2} (m_{11} + m_{22}) = \cos \Phi + a \sin \Phi \), and \( a = \)
For an array with \( N + 1 \) resonators, we define the reflection efficiency \( R \) as the ratio \( S_{-1}/S_1 \), which can be found from the condition \( S_{N+1} = 0 \),

\[
R = \left| \frac{M_{12}}{M_{11}} \right|^2 = \frac{a^2 A^2}{(a^2 + 1)A^2 + B^2 - 2ABC}.
\]  

(6)

where \( A = \sin((N + 1)\theta) \), \( B = \sin(N\theta) \) and \( C = \cos \theta \). It can be seen that the reflection spectrum is determined by four parameters, i.e., the resonant frequency \( \omega_0 \), the quality factor \( Q \), the phase shift \( \Phi \), and the resonator number \( N \).

First the effect of the phase shift is considered. The reflection spectra of two side-coupled resonators with the different phase shifts \((\pi/4, \pi/2, \pi)\) are shown in Fig. 3a. It can be seen that when the phase shift is equal to \( n\pi \) or \((n + 1/2)\pi\), the reflection is smooth and symmetric. Moreover, the flat-top can be obtained in the case of \((n + 1/2)\pi\). From (6) it can be proven that if \( \Phi = n\pi \) or \((n + 1/2)\pi\), the spectrum is symmetric \([4, 5]\). Therefore the phase shift is chosen to be \( n\pi \) or \((n + 1/2)\pi\).

When all the resonators are uniform and have the same phase shift of \((n + 1/2)\pi\), there are sidelobes due to the optical interference between the resonators when the resonator number \( N \geq 3 \)[4, 6], as shown in Fig. 3b. From (6), the reflection efficiency can be rewritten as

\[
R|_{\Phi=(n+1/2)\pi} = \frac{a^2 \sin^2[(N + 1)\theta]}{1 - a^2 + a^2 \sin^2[(N + 1)\theta]}.
\]  

(7)

From (7) (let \( R = 1 \)), the width of the flat-top is equal to \(2\omega_0/(Q \Delta \omega)\). Therefore, a lower \( Q \) would lead to a broader flat-top. Conceptually, \( Q \) represents the coupled coefficient between the resonator and the waveguide. Low \( Q \) means the strong couple, and therefore the broad resonance peak \([8]\). Figure 3c compares two cases where \( Q = 500 \) and \( Q = 1000 \), the phase shift are both \( \pi/2 \). It is obvious that the flat-top width of the former resonance is wider.

The number of the sidelobes is determined by the zeros of \( R \), which has the same zeros with \( \sin((N + 1)\theta) \) in (7). If \((N + 1)\theta = m\pi\), then \( \sin((N + 1)\theta) = 0 \); if one period of \( \theta \)
Phase shift \( N = \omega_2 \text{INT} \left( \frac{\pi}{2}, \right) \); (dotted line) \( N_A = \omega_2 \) and \( N \rightarrow A = 2 \) (5) (INT, 5), can be formed by only two side-apodization [9, 10], but this technique is not practical due to the discrete symmetry of the photonic crystal. Therefore, we propose two optimization techniques to suppress the sidelobes.

The first optimization technique is the chirp technique. It main feature is to change the phase shift linearly and break the interference. As a result, the sidelobes can be suppressed effectively within 10 resonators. According to the CMT, the phase shift should be \((n/2) \pi\) to maintain the symmetry of the reflection spectrum (Fig. 3b). Therefore, the phase shift of some resonators is replaced by \(n \pi\) instead of \((n + 1/2) \pi\) to maintain spectral symmetry while achieving the flat-top. Consider that the large phase shift results in long device and high loss in practice while the small phase shift results in direct coupling between the resonators (which induces multimode) [6], the resonators with the phase shifts of \((5/2) \pi\) and \(3\pi\) are chosen. The optimization details can be seen in Table 1. For example, when \(N = 6\), the phase shift is \([5\pi/2, 3\pi, 5\pi/2, 3\pi, 5\pi/2]\), the reflection spectrum is shown in Fig. 4. Compared with the spectrum of the uniform phase shift (dashed line), the reflection spectrum using the chirp technique (solid line) has flat-top and steep roll-off. The sidelobes are completely suppressed.

Another optimization technique is the cascading technique. It uses many resonator elements with the different resonant frequencies in series [12]. According to the above analyses, the basic structure can be formed by only two side-coupled resonators, which is called a resonator element since in Fig. 3b the sidelobes disappear for \(N = 2\). The phase shift is set to \(5\pi/2\) and the quality factor is chosen to be low to obtain a broad flat-top. By carefully designing the quality factor and the resonator frequency, the filter can obtain the arbitrary bandwidth. When many such resonator elements with the different resonance frequencies are cascaded, a broad bandwidth

### Table 1: The Chirp Optimization Technique

<table>
<thead>
<tr>
<th>( N )</th>
<th>Phase shift ( \Phi )</th>
<th>( N )</th>
<th>Phase shift ( \Phi )</th>
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<td>4</td>
<td>( \left[ \frac{5\pi}{2}, 3\pi, \frac{5\pi}{2} \right] )</td>
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<tr>
<td>5</td>
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<td>6</td>
<td>( \left[ \frac{5\pi}{2}, 3\pi, 3\pi \right] )</td>
</tr>
<tr>
<td>7</td>
<td>( \left[ \frac{5\pi}{2}, 3\pi, 3\pi, \frac{5\pi}{2}, \frac{5\pi}{2} \right] )</td>
<td>8</td>
<td>( \left[ \frac{5\pi}{2}, 3\pi, \frac{5\pi}{2}, 3\pi, \frac{5\pi}{2} \right] )</td>
</tr>
<tr>
<td>9</td>
<td>( \left[ \frac{5\pi}{2}, 3\pi, 3\pi, \frac{5\pi}{2}, \frac{5\pi}{2}, 3\pi \right] )</td>
<td>10</td>
<td>( \left[ \frac{5\pi}{2}, 3\pi, 3\pi, \frac{5\pi}{2}, 3\pi, \frac{5\pi}{2} \right] )</td>
</tr>
</tbody>
</table>

**FIGURE 4** Reflection spectra of the six-resonator reflection filter with (dashed line) and without chirp (solid line)

**FIGURE 5** Reflection spectra after using the cascading technique. (a) \( N = 2 \) and \( \Phi = \pi/2 \), with different resonance frequency \( \omega_0 = 0.367 \) (solid line), \( \omega_0 = 0.3675 \) (dashed line), \( \omega_0 = 0.368 \) (dotted line); (b) reflection spectra of the six-resonator reflection filter with (solid line) and without cascading (dashed line)
filter without the sidelobes can be obtained because the resonator elements are not uniform. For example, in Fig. 5a the frequencies of three side-coupled resonators are 0.367, 0.3675 and 0.368, respectively (the quality factor is kept the same); the individual pass-band overlaps at some point, which results in a broad pass-band. As shown in Fig. 5b, the new filter has flat-top without the sidelobes. The advantages are obvious compared to the original filter formed by six identical resonators. In the same way, the broad pass-band filters can be implemented by overlapping the individual reflection peaks in a continuous manner.

3 Simulation results and discussions

To verify the theoretical model developed in Sect. 2, we will compare the spectra of the filters optimized by the chirp technique and the cascading technique with the simulation results given by FDTD. In practical designs of the PhC reflection filter, a two-dimensional (2D) photonic crystal structure which consists of a square lattice of dielectric rods is considered. The dielectric constant and the radius of the dielectric rods are 11.56 and 0.2 \( a \) (\( a \) is the lattice constant of the square lattice), respectively. The waveguide is formed by removing one row of the dielectric rods. The single mode resonator is formed by changing the radius of the rod.

Following the design in Fig. 1, six identical resonators are periodically spaced along the PhC waveguide direction, the distance between any two neighboring resonators is 5\( a \). The radius of the resonator is 0.048\( a \). In order to obtain low quality factor, the distance between each resonator and the waveguide is chosen to be 0\( a \). As discussed in Sect. 2, the phase shift \( \Phi \) is a critical parameter in the design of the multi-resonator reflection filter. It can be defined as

\[
\Phi = \frac{n_{eff} L}{c},
\]

where \( n_{eff} \approx c dk / d \omega \) is the effective refractive index [14], which can be obtained easily from the dispersion relationship of the guided mode of the waveguide. In order to obtain a flat-top reflection spectrum with the sharp roll-off, the phase shift \( \Phi \) is chosen to be \( (n + 1/2)\pi \) as discussed in Sect. 2. According to (8), when the integer \( n \) is set at 0, 1, 2 and 3, respectively, the distance \( L \) and the normalized frequency \( a / \lambda \) have the relationship shown in Fig. 6a and b. At a particular frequency of 0.368, the value of \( L \) is always integer for different integer \( n \). Hence, the resonance frequency is chosen to be 0.368 in

![FIGURE 6](image_url) Distance along waveguide determined by (a) \( \Phi = (n + 1/2)\pi \); (b) \( \Phi = n\pi \)

![FIGURE 7](image_url) Reflection spectra of side-coupled resonators filter. (a) Simulated by using the CMT (dashed line) and the FDTD (solid line); (b) simulated by the FDTD with (solid line) and without chirp (dashed line)
order to ease the control of the phase shift. When $L$ is odd it has $\Phi = (n + 1/2)\pi$ and when $L$ is even it has $\Phi = n\pi$.

The reflection spectra simulated using FDTD are shown in Fig. 7. As to the optical system depicted in Fig. 1, the reflection spectrum gives the sidelobes on both sides of the reflection peak. The solid curve and the dashed curve in Fig. 7a represent the filter’s reflection calculated by CMT and FDTD, respectively. It can be seen that the outcome of CMT agrees well with FDTD. The asymmetry of the sidelobes is due to the difficulty to control the phase shift precisely. The time step is also an important parameter that affects the resolution of the spectra.

Figure 7b compares the reflections simulated by FDTD for the uniform and the chirp filter. In the chirped system, the phase shift $\Phi$ is set as $[5/2\pi, 3\pi, 5/2\pi, 3\pi, 5/2\pi]$. Therefore, the distance between two neighboring resonators is $6\alpha$ with the phase shift $\Phi = 3\pi$. By then, the sidelobes are completely suppressed except the ripples which are due to the finite size of the photonic crystal – the abruptly beginning and ending of the waveguide [15].

To verify the cascading technique, two identical resonators form a resonator element. Three such elements with different resonance frequencies are then cascaded. The reflection spectrum is shown in Fig. 8. The resonance frequencies are 0.3682 (dashed line), 0.3689 (dotted line), 0.3696 (dash-dotted line), respectively. The solid line is the result of the FDTD simulation. It can be seen that there is no sidelobe on both sides when the optimized cascading structure is used.

From Figs. 7a and 8, it can be seen that a good agreement between CMT and the FDTD simulation is achieved, but it does not involve the time consuming computation. It also proves the effectiveness of the optimization techniques to suppress the sidelobes. The structures are easy to fabricate. The difference between two techniques is that the cascading technique is more flexible in controlling the bandwidth whereas the chirp has steeper roll-off.

4 Conclusions

In this paper, by using the CMT and the TMM, the side-coupled resonators are discussed. If the phase shifts of the resonators are different from each other, the interference between the resonators breaks down and the sidelobes in the reflection response are suppressed. From the viewpoint of the fabrication, it is a simple way to implement the high performance filter. We also proposed another technique which cascades two-side-coupled resonators with different resonance frequencies. Theoretical analysis shows that it also shows good performance when it used to implement a filter. Moreover the bandwidth can be controlled by selecting a proper number of side-coupled resonators in the structure.

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