Chaotic dynamics of a passively mode-locked soliton fiber ring laser

L. M. Zhao, D. Y. Tang, and A. Q. Liu
School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, 639798

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We report on the experimental and numerical studies of the chaotic dynamics of a soliton fiber ring laser passively mode-locked by using the nonlinear polarization rotation (NPR) technique. Period-doubling route to chaos on the soliton repetition rate of either the single pulse soliton or the bound solitons of the laser was experimentally observed. Based on a coupled complex Ginzburg-Landau equation model and also taking into account the laser cavity effect, we further show numerically that the period-doubling bifurcations and route to chaos are intrinsic properties of the laser, whose appearance is independent of the details of the laser cavity design and the laser soliton operation. Property of the solitons under the dynamical bifurcations is also numerically investigated.

Since Ikeda first showed that the transmission of a passive nonlinear optical ring cavity can exhibit multistability and chaos in response to a constant incident light, extensive studies on the dynamic property of both the passive and active nonlinear optical ring cavities have been done. However, the majority of research was focused on the continuous-wave operation of the cavities. It would be of interest to investigate the dynamical features of the cavities and their manifestations under the ultrashort pulse operation, in particular when the ultrashort pulse is itself an optical soliton. The present paper presents the experimental and numerical studies on such a system. It is shown that even though the circulating light in an active fiber ring cavity corresponds to optical solitons, which are inherently stable optical pulses, the cavity can still exhibit a period-doubling route to chaos on the soliton repetition rate.

I. INTRODUCTION

Passively mode-locked soliton fiber ring lasers, as an alternative source of the ultrashort optical pulses, have been extensively investigated.1–4 By taking advantage of the fiber Kerr nonlinearity and anomalous dispersion, optical soliton is automatically formed in the lasers. Soliton operation further suppresses the mode-locked pulse width and leads to the generation of ultrashort pulses. Conventionally, soliton operation of the fiber lasers is modeled by the extended nonlinear Schrödinger equation (NLSE) or the complex Ginzburg-Landau equation (GLE), which takes into account the interaction between the fiber nonlinearity and dispersion, gain and losses. Although due to the laser gain and output loss an optical pulse circulating in the laser cavity periodically changes its parameters, it was found that the averaged properties of the pulse still follow the NLSE or the GLE. In particular, at a fixed position in the cavity the pulse profiles are invariant with time.5 Various applications of the passively mode-locked fiber soliton lasers have been proposed, such as in the optical signal processing and fiber optical communication systems.

Although it was generally believed that soliton pulses from a passively mode-locked fiber laser are stable, recent theoretical and experimental studies on the lasers have revealed several kinds of soliton peak power fluctuations, e.g., Zhao et al. have shown that due to the interaction between the nonlinear polarization rotation of light and the existence of a polarizer in the cavity, the soliton pulse train formed in the lasers could exhibit a low frequency peak power modulation.6 Nevertheless, such a soliton peak power modulation could be avoided by appropriately setting the linear cavity phase delay bias of the laser. On the other hand, a soliton fiber laser is in essence a nonlinear dynamical system, where a high peak power optical pulse circulates in a nonlinear ring cavity. It was first shown by Ikeda that transmission of a passive nonlinear ring cavity could exhibit chaotic behavior in response to a constant incident light.7 Triggered by Ikeda’s result, extensive theoretical and experimental studies on the dynamic features of passive and active nonlinear ring cavities have also been carried out.8–11 In particular, it was shown that optical instabilities and chaos exist in the periodically forced light pulse propagation in the passive nonlinear ring cavity.12,13 In addition, recent experimental studies on the mode-locked lasers, such as in an additive-pulse mode-locked laser14 and femtosecond Ti:sapphire lasers,15,16 have also revealed period-doubling of the mode-locked pulses in the lasers, which demonstrated that short pulse propagation in the active nonlinear ring cavity can also exhibit chaotic dynamics. However, to the best of our knowledge, the chaotic soliton dynamics of the soliton fiber ring lasers is so far not addressed. As a soliton pulse is a stable nonlinear wave, how would a nonlinear optical ring cavity affect the soliton propagation, and whether the soliton feature of an optical pulse could prevent the appearance of chaotic dynamics of the cavity are still open questions.

We have experimentally investigated the soliton dynamics of a passively mode-locked soliton fiber ring laser with dispersion-managed cavity,17 and observed period-doubling bifurcations and period-doubling route to chaos of the single-pulse soliton18 and the bound solitons.19 Our experimental
results clearly showed the deterministic nature of the soliton dynamics. However, due to the ultrashort pulse width of the solitons and the limitation of our measurement devices, apart from the energy variation of the soliton pulses and the time-averaged soliton spectra, details of the soliton features under the dynamical bifurcations could not be experimentally investigated. To have a better understanding on the chaotic soliton dynamics of the laser, and also gain an insight into the soliton property under dynamical bifurcations, we have further conducted numerical simulations on our laser system. In this paper we present results of our simulations. For the sake of comparison, we have also briefly reviewed our experimental setup and results in the current paper. Our numerical simulations have not only revealed the soliton period-doubling bifurcations and period-doubling route to chaos of our fiber laser, but also shown that the appearance of the phenomenon is independent of the laser cavity design. A soliton period tripled state and the period-doubling of the state were also numerically obtained in our simulations, which confirms the experimental observation of Tamura et al. on a period-tripled soliton laser emission state.20 In addition, based on the numerical simulations we have also studied soliton features under the period-doubling bifurcations. It further complements our experimental measurement to the soliton dynamics in the lasers.

II. EXPERIMENTAL SETUP AND RESULTS

Our experimental setup is shown in Fig. 1. The fiber laser has a ring cavity of about 12 meters. The cavity comprises a two-meter-long 8000 ppm erbium doped fiber with a group velocity dispersion of about +70 ps/nm/km to achieve dispersion-managed cavity, and 10-meter standard single mode fiber (SM28) with group velocity dispersion of about −18 ps/nm/km. An isolator is inserted in the cavity to force the unidirectional operation of the laser. The nonlinear polarization rotation (NPR) technique is used to achieve the self-started mode locking.21 To this end two polarization controllers, one consisting of two quarter-wave plates and the other two quarter-wave plates and one half-wave plate, are used to adjust the polarization of the light. A cubic polarization beam splitter is used to output the laser pulses. The polarization controllers and the beam splitter are mounted on a 7 cm long fiber bench. The laser is pumped by a high power Fiber Raman Laser source (BWC-FL-1480-1) of wavelength 1480 nm, and the soliton propagation in the laser cavity is monitored with an optical spectrum analyzer (Ando AQ-6315B), a 26.5 GHz rf spectrum analyzer (Agilent E4407B ESA-E SERIES) and a 350 MHz oscilloscope (Agilent 54641A) together with a 5 GHz photodetector. A commercial optical autocorrelator (Autocorrelator Pulsescope) was used to measure the pulse width of the soliton pulses.

With the experimental setup it was found that when the orientations of the waveplates were appropriately set, the peak power of the soliton pulses formed in the cavity could reach a very high value. Consequently, the strength of the nonlinear interactions between the soliton pulses and the cavity components, such as the optical fibers and the gain medium, becomes strong. To a certain level of the nonlinear interactions it was observed that the soliton energy pattern of the laser output experienced period-doubling bifurcations and route to chaos. Figure 2 shows for example an experimentally measured period doubling route to chaos of the single-pulse soliton of the laser. The result was obtained with fixed linear cavity phase delay bias, but the pump power was gradually increased. At a relatively weak pump power, a stable soliton pulse train with uniform pulse energy was obtained. The pulses repeated themselves with the cavity fundamental repetition rate [Fig. 2(a)]. Carefully increasing the pump power to a certain value, the intensity of the soliton pulse suddenly became nonuniform as shown in Fig. 2(b). In the state the soliton energy alternated between two values. It is to see that although the round trip time of a soliton circulating in the cavity is still the same, the soliton energy returns back to its previous value only after every two round-trips, forming a so-called period-doubled state as compared with that of Fig. 2(a). Further slightly increasing the pump power, a period-quadrupled state then appeared [Fig. 2(c)]. Eventually the period-doubling process ended up at a chaotic soliton pulse energy variation state [Fig. 2(d)]. The same period-doubling route to chaos has also been observed on the bound solitons, where the total energy of the bound solitons exhibited period-doubling bifurcations and route to chaos.19 In the case of a bound-soliton several solitons bind together with close soliton separations. Although the state of a bound-soliton could be easily identified by its optical spectrum and...
autocorrelation trace, experimentally it is difficult to monitor
the energy variation of each individual soliton within a
bound-soliton. Therefore, experimentally no detailed infor-
mation on property of each individual soliton during a
period-doubling bifurcation process could be obtained.
Period-doubling bifurcations of bound solitons with different
soliton separations were also experimentally obtained.

III. NUMERICAL SIMULATIONS

The soliton dynamics of the laser is determined by the
soliton circulation in the nonlinear ring cavity and the peri-
odical interaction with the cavity components. Therefore,
in order to simulate the soliton dynamics, in particular to gain
an insight into the detailed soliton property in the various
period-doubled states, we have used a so-called round-trip
model to simulate the soliton evolution in our laser. Using
the same model we have previously successfully modeled
other soliton properties of the passively mode-locked fiber
ring lasers such as the subsideband generation, soliton peak
nonuniformity, and soliton sideband asymmetry. Concre-
tely, we start our simulation with an arbitrary small light
pulse and let it circulate in the cavity. In accordance with
the laser configuration, the light will first pass through the polari-
zation and the polarization controller, then propagate consecu-
tively in the single mode fiber (SMF), the erbium-doped fiber
(EDF) and the other piece of SMF, subsequently through the
output coupler (beam splitter). The light propagation in the
optical fibers is described by the extended coupled complex
nonlinear Schrödinger equations

$$\frac{\partial u}{\partial z} = i \beta u - \frac{\partial u}{\partial t} - \frac{i k'' \sigma^2 u}{2 \sigma^2} + \frac{i k''' \sigma^3 u}{6 \sigma^3}$$

$$+ i \gamma \left( |u|^2 + \frac{2}{3} |v|^2 \right) u + \frac{i \gamma}{3} \sigma^2 v^* + \frac{g}{2} u + \frac{g}{2} \frac{\sigma^2 u}{\Omega},$$

where \( u \) and \( v \) are the normalized envelopes of the optical
pulses along the two orthogonal polarized modes of the op-
tical fiber. \( 2\beta=2\pi\Delta n/\lambda \) is the wave-number difference
between the two modes. \( 2\delta=2\beta \lambda/2\pi c \) is the inverse group
velocity difference. \( k'' \) is the second-order dispersion coeffi-
cient, \( k''' \) is the third-order dispersion coefficient, and \( \gamma \)
represents the nonlinearity of the fiber. \( g \) is the saturable gain
coefficient of the fiber and \( \Omega \) is the bandwidth of the laser
gain. For undoped fibers \( g=0 \); For erbium doped fiber, we
have further considered its gain saturation as

$$g = G \exp \left[ - \frac{f(|u|^2 + |v|^2)dt}{P_{sat}} \right],$$

where \( G \) is the small signal gain coefficient and \( P_{sat} \) is the
normalized saturation energy. Whenever the light meets a
discrete cavity component, such as the polarizer and the polari-
zation controller, the light field is multiplied by the
transfer-matrix of the discrete cavity component. The result
of the previous round of calculation is then used as the input
of the next round of calculation until a steady state
is reached. To make our simulation possibly close to the
experimental situation, we used the following parameters for
our simulations: \( \gamma=3 \text{ W}^{-1} \text{ km}^{-1} \), \( k''=0.1 \text{ ps}^2/\text{nm/km} \), \( \Omega_g=25 \text{ nm} \), beat length \( L_b=L/2 \), and the orientation of the
intracavity polarizer to the fiber fast birefringent axis
\( \Psi=0.125\pi \), cavity length \( L=6\text{SMF}+2\text{EDF}+4\text{SMF}=12 \text{ m} \) and
gain saturation energy \( P_{sat}=250 \). To simulate the feature of
cavity dispersion management we have used fiber group ve-
clocity dispersion (GVD) as \( k''_{EDF}=50 \text{ ps/\text{nm/km}} \), and

FIG. 2. Period-doubling route to chaos of the soliton trains. (a) Period-1 state; (b) period-2 state; (c) period-4 state; (d) chaotic state.
With the above laser parameter selection, the laser can achieve self-started mode-locking in the linear cavity phase delay bias range of \( \pi < \delta \phi < 2\pi \). Numerically we found that with too small linear cavity phase delay bias selection, the peak power of the mode-locked pulses was clamped by the polarization switching effect of the cavity. No soliton could be formed in the laser. With the linear cavity phase delay bias set in the range of \( 1.2 \pi < \delta \phi < 1.5 \pi \), conventional soliton operation as observed experimentally could always be obtained. With our laser parameter selection, increasing the linear cavity phase delay bias \( \delta \phi \) increases the nonlinear polarization switching threshold of the cavity. Consequently, the solitons formed in the cavity have higher peak power. When the peak power of the solitons becomes strong enough, they then experience period-doubling bifurcations or period-doubling route to chaos. Figure 3 shows, for example, the soliton period-doubling route to chaos numerically obtained when the linear cavity phase delay bias was set as \( \delta \phi = 1.6 \pi \). With the linear cavity phase delay setting, at the low pump intensity the output of the laser is a uniform soliton train. As the pump power is increased, to a certain value the soliton peak intensity suddenly changed to alternating between two values, exhibiting a so-called period-doubling bifurcation. This period-doubling bifurcation occurred again as the pump strength is further increased. Eventually the soliton pulse output became chaotic as shown in Fig. 3(d). Note that the maximum soliton peak power grew up with the increase of the pump strength. Extensive numerical simulations have demonstrated that the period-doubling bifurcations could actually occur under different laser cavity parameter settings, provided that the soliton peak power was unclamped by the nonlinear polarization switching effect of the cavity. Nevertheless, in order to obtain a full route to chaos, the cavity parameters such as the fiber dispersion and fiber lengths must be appropriately selected. In one of our numerical simulations even the period doubling route to chaos of a period-three state, which in the nonlinear dynamics theory is known as a periodic window within the chaotic regime, has also been obtained as shown in Fig. 4. To obtain the state we have used \( k'_{\text{EDF}} = 70 \text{ ps/nm/km}, k'_{\text{SMF}} = -20 \text{ ps/nm/km}, \) and the linear cavity phase delay bias \( \delta \phi = 1.5 \pi \).

With multiple solitons coexisting in the laser cavity, experimentally it was shown that the solitons could interact with each other and form the so-called bound states of solitons. In our numerical simulations bound states of solitons have also been obtained. In fact our previous numerical studies on the soliton interaction in the passively mode-locked fiber lasers have shown that, due to the existence of mode-locking in the laser, solitons have the tendency of forming bound states under direct soliton interaction. Period-doubling bifurcations and route to chaos have also been numerically revealed for the bound solitons. Figure 5 shows for example one of such results obtained. In calculating the state we used the same laser parameters as those for obtaining the single-pulse soliton period-doubling route to chaos, only the pump strength and initial state were different. Figure 5(a) shows that two solitons coexist in the cavity and bind together. Note that due to the close separation between the solitons, their optical spectra have strong

**FIG. 3.** Soliton period-doubling route to chaos numerically calculated. (a) Period-1 state, \( G=800 \); (b) period-2 state, \( G=850 \); (c) period-4 state, \( G=902 \); (d) chaotic state, \( G=915 \).
intensity modulations, but from round to round the modulation patterns do not change, which indicates that the phase difference between the solitons is fixed as well. The binding nature of the solitons is represented by the fixed soliton separation and phase difference even under the existence of soliton interaction between them. Increasing pump strength from the state of Fig. 5(a), both solitons experienced simultaneously period doubling bifurcations and route to chaos. Associated with the soliton period-doubling the soliton separation between the bound solitons also changed slightly. However, after the period-doubling bifurcation the soliton separation then remained constant again. This soliton separation change suggests that the dynamic bifurcation of the system could affect the soliton interaction.

Based on the numerical model we have further investigated the soliton features in each of the period-doubled states. Figure 6 shows, for example, the soliton profiles and corresponding optical spectra of the laser output in the period-2 state. In this state the soliton energy alternates between two values, which were also demonstrated experimentally. Numerical simulations further show that actually all the parameters of the soliton, such as the pulse profile, pulse width, and peak power, vary within one period in the laser cavity. It is well known that the soliton formed in a laser is an average soliton. A soliton circulating in the cavity periodically changes its parameters due to the action of gain and output loss. In a stable situation the gain experienced by the soliton within one round trip balances the losses, therefore, after one round trip the soliton returns to its previous state. Soliton lasers operating in this situation emits stable uniform soliton pulse train. However, solitons emitted in the above situation also have weak pulse energy and peak power. Comparing with the period-1 state, soliton in the period-2 state still has the feature: Within one period the soliton parameters vary, however, the period is no longer equals the natural cavity length but twice of it. The ultimate soliton peak power

![FIG. 4. Soliton profiles and corresponding optical spectra numerically calculated. (a) and (d) State of period-3, $G=730$; (b) and (e) state of period-6, $G=735$; (c) and (f) chaotic state, $G=750$.](image_url)
and pulse width that a soliton can reach in the state is also substantially higher and narrower than those of the soliton in the period-1 state. It is probably because the soliton needs to reach a higher peak power that it has to go through the cavity twice in order to complete one period. It is interesting to compare the soliton spectra of the laser output after each round trip. While after one round trip (or half of the period) the soliton spectrum has reasonably the same feature as that of the soliton in the period-1 state, after another round trip the soliton spectrum then becomes very different. Extra soliton sidebands clearly appeared in the spectrum. How these extra sidebands link to the soliton parameter variation and the cavity periodicity needs to be further studied. Nevertheless, the relation of the new sideband generation with the

FIG. 5. Period-doubling route to chaos of bound solitons. (a) and (e) state of stable bound solitons, \( G=1149 \); (b) and (f) state of period-2 of the bound solitons, \( G=1300 \); (c) and (g) state of period-4 of the bound solitons, \( G=1353 \); (d) and (h) chaotic state of the bound solitons, \( G=1358 \).
intrinsic soliton dynamics is obvious. Figure 7 shows the soliton pulse profile and spectrum variation within the period-4 state. As it is essentially a further period-doubling of the period-2 state, therefore, overall the soliton spectra look like a duplication of those of the period-2 state. However, we point out that careful comparison with those of the period-2 state shows that further new spectral sidebands have been generated. The spectra shown in Figs. 7(f) and 7(h) are also different.

To understand the soliton features observed, we note that when the soliton energy becomes strong, cavity nonlinear effect would become important. A soliton circulating in the nonlinear laser cavity is in essence a kind of nonlinear mapping. It is well-known that the nonlinear mapping can generate deterministic chaos including the period-doubling bifurcations and route to chaos. It is, therefore, not surprising that under appropriate cavity conditions, soliton in the laser can experience period-doubling route to chaos. As the dynamics is physically a manifestation of the nonlinear cavity effect, the appearance of the period-doubling phenomenon itself is independent on whether the light circulating in the cavity is a soliton or not, and whether the soliton is a single-pulse soliton or bound solitons. Finally, we point out that based on the same numerical model we have also obtained period-doubling bifurcations and period-doubling route to chaos in the nondispersion-managed fiber lasers. Similar property and features of solitons under dynamical bifurcations have been obtained. This numerical result further supports that the period-doubling/tripling bifurcation and route to chaos is an intrinsic property of the fiber soliton laser. Its appearance is independent of the details of laser cavity design and the soliton property.

IV. CONCLUSIONS

In conclusion, we have both experimentally and numerically investigated the period-doubling bifurcations and period-doubling route to chaos of the single-pulse soliton and bound solitons in a passively mode-locked fiber soliton ring laser with dispersion-managed cavity. The laser was mode-locked by using the nonlinear polarization rotation technique. Experimentally, we found that when the peak power of solitons in the cavity is strong, after one round-trip in the cavity the soliton energy does not return to its previous value, but returns to the previous value after two or four round trips. Under strong pumping the soliton energy even does not return to its previous value at all. This soliton energy variation follows exactly the so-called period-doubling route to chaos of the nonlinear dynamics systems, which suggests that solitons in the laser can also exhibit deterministic chaos dynamics. Based on the well-known round-trip model we have numerically confirmed the dynamics of the

FIG. 6. Soliton pulse profiles and corresponding optical spectra in a period-2 state.
FIG. 7. Soliton pulse profiles and corresponding optical spectra in a period-4 state.
laser and identified its conditions of appearance. Our numerical simulation also revealed that in a period-doubled state, the soliton peak power can become very high and soliton pulse width becomes very narrow. This property of the solitons may be exploited to get even narrow pulse width and higher peak power optical pulses. In addition, our simulation result shows that the dynamic variation of the soliton energy introduces new sidebands in the soliton spectra and the dynamic bifurcation of the system can slightly change the separation of the bound solitons.