A modeling and analysis of spring-shaped torsion micromirrors for low-voltage applications

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Abstract

An electrostatic torsion micromirror is designed using the optimized spring-shaped torsion beams and U-shaped groove supporters. The main advantages of this design will be a reduction in the pull-in voltage for low-voltage applications and the function as a switch of the instability mode by adjusting the effective bending stiffness of the torsion beam. In this design, a theoretical model is developed to demonstrate the static characteristics of the electrostatic torsion micromirror. The pull-in effect is investigated specifically to predict the pull-in voltage, pull-in angle, and pull-in displacement. These parameters depend significantly on the electrode size and position, position of the groove, and ratio of the bending and torsion effect of the torsion beam. In addition, the effective torsion and bending stiffness model is provided using the energy method with the objectives to optimize the spring-shaped geometries of the torsion beams and to decrease the pull-in voltage below 2 V. The U-shaped groove supporters are applied in the theoretical model to adjust the effective bending stiffness of the torsion beam and to switch the instability mode of the torsion micromirror. The theoretical analysis is consistent with the numerical simulation using MEMCAD and experimental results measured by an optical projection method.

Keywords: Electrostatic torsion micromirror; Pull-in effect; Optical MEMS; MOEMS

1. Introduction

Microoptoelectromechanical systems (MOEMS) which include optical cross-connects [1,2], optical switches [3], microscanners [4], digital micromirror device (DMD) [5], electrically controlled variable attenuators [6], etc., have been widely investigated in the telecommunications networks [7]. Electrostatic torsion micromirrors, which possess good dynamic response and small possibility of adhesion, have been widely used in the applications of many of these optical MEMS systems [2,8–14]. These applications include digital projection displays [8,9], spatial light modulators [10,11], optical crossbar switches [12], and adaptive optics [13].

In the past several years, the characteristics of torsion micromirrors have been extensively studied [2,12,14–24]. An important property of the electrostatic torsion micromirrors is their pull-in characteristics, where the electrostatic force/torque overcomes the mechanical force/torque and the movable plate snaps abruptly to the fixed electrode plate, when the applied voltage on the movable plates is increased above a certain voltage. The specific pull-in parameters (pull-in voltage, pull-in angle, and pull-in displacement), which are determined by the geometrical design of the micromirror and actuating electrodes, play an important role in influencing the performance of the torsion micromirror.

Previous models merely considered the torsion micromirror as one degree-of-freedom (1-DOF) actuator. The effect of the dimension and location of the bottom electrode plate was analyzed [2,12,14–24]. The effect of the non-linear strain stiffening of the torsion beam was also
considered [17,18]. These cases will be efficient only when the vertical displacement of the torsion micromirror is significantly small. However, when the vertical displacement of the torsion micromirror is of the same order to the torsion angle, the torsion and bending effect are coupled to affect the static characteristics of the torsion micromirror. As a result, this pull-in torsion angle using previous models may be inaccurate. In addition, when the vertical displacement of the torsion beam reaches 10% of the gap between the micromirror which works as optical mirror and electrode plate (about 0.5 μm), the phase of reflected light beam will be shifted by half or even one wavelength of the lights, and thus seriously affect the design and usage of relevant optical applications [10,11]. Therefore, accurate analysis for the torsion angle and vertical displacement of the torsion micromirror is very important to the optical devices and applications.

The critical pull-in voltage plays an important role in the consideration for practical usages. The torsion micromirror normally operates below 5 V in order to be compatible with IC components. In general, the pull-in voltage should be as low as possible to prevent electrical breakdown. As a result, it is necessary to use the effective torsion and bending model to optimize the geometry of the torsion beam and to reduce the pull-in voltage. In addition, the pull-in phenomenon can be controlled within an accepted range using the U-shaped groove supporters.

In this paper, the pull-in characteristics of a torsion micromirror are investigated using the coupled model. The coupled model provides a pull-in instability mechanisms, which is dominated by the torsion or bending effect of the torsion micromirror. The effective torsion and bending stiffness model is obtained using the energy method. This is derived to optimize the spring-shaped geometries of the torsion beams and to reduce the pull-in voltage. The U-shaped groove supporters are considered in the theoretical model to adjust the effective bending stiffness of the torsion beam and to switch the instability mode of the torsion micromirror.

2. Theoretical model

A schematic diagram of a torsion micromirror and its cross-sectional view are shown in Figs. 1(a) and (b), respectively. The micromirror is supported by two torsion beams, which are mounted to two fixed anchors on the substrate. The torsion beams are particularly designed as spring-shaped geometries to reduce the applied voltage. U-shaped grooves are used to change the motion state of the torsion micromirror as supporting blocks. Beneath the micromirror, there are two electrodes on the substrate. The micromirror can be driven to bend down and to rotate by the applied voltage between the micromirror and electrode plate. In Fig. 1, L, a, and t represent the length, width, and thickness of the micromirror, respectively; l denotes the length of the torsion beam between the micromirror and groove; h is the gap between the micromirror and electrode; and $h_g$ is the gap between the torsion beam and groove. $a_1$ and $d_2$, which denote the inner and outer distance between the two electrodes, define the size and position of the electrodes.

2.1. Effective torsion and bending stiffness

This section presents a mechanical model of the effective torsion and bending stiffness. This is used to study how the pull-in voltage of the torsion micromirror depends on the geometrical parameters and material properties. The torsion micromirror can be simplified as a fixed-fixed beam, which is shown in Fig. 2. The x and y axes are assumed to be parallel to the length and width of the micromirror, respectively, and the z-axis is directed upwards, perpendicular to the substrate. $L_1$ is the length of the torsion beam between the anchor and spring whereas $L_2$ is the length of the torsion beam between the micromirror and spring. $w_i$ ($i = 1, 2, 3, 4$) is the width of each different section of the torsion beam. $H$, $S$, and $n$ are the height, width, and number of the spring, respectively. $P_0$, $M_0$, and $T_0$ are the constraint reacting force, moment, torque of the anchor, respectively. The selection of a functional form is interesting as it allows the extension of this model to other beam structures.

Energy method [25] is used to find displacements and to analyze structures. When the energy method is applied to derive the mechanical model for the effective stiffness of the structures that behave linearly and for which the principle of superposition applies, the complementary energy equals to the strain energy of the structure. Both quantities are expressible as quadratic functions of loads. Therefore, the strain energy, $\Pi$, of the torsion micromirror is given

$$\Pi = \int_{f} \frac{M^2}{2EI_0} d\xi + \int \frac{T^2}{2GI_p} d\xi,$$

where $f$ denotes the whole configuration of the torsion micromirror; $E$ is the Young’s modulus of the torsion beam; $G = E/(1 + \nu)$ is the shear modulus of the torsion beam; $\nu$ is the Poisson’s ratio of the torsion beam; $M$ and $T$ are the bending moment and torque of the torsion beam, respectively. $I_b$ and $I_p$ are the axial and polar moment of inertia of the rectangular cross-section, respectively.

Only displacement from the torsion and bending is considered in this analysis. Deformation from beam elongation and beam shortening is neglected. Castigliano’s second theorem [25] is used in this case to determine the torsion angle and vertical displacement of the structures. The theorem states that at the point of application

$$\delta = \frac{\partial \Pi}{\partial P}, \phi = \frac{\partial \Pi}{\partial M} \text{ and } \theta = \frac{\partial \Pi}{\partial T},$$

where $P$ is the applied force; $\delta, \phi, \theta$, are the corresponding vertical displacement, angular displacement, and torsion angle, respectively. The effective bending
stiffness of the torsion beam, $K_{\text{eff}}$, is derived by dividing the applied force over the center of the structure, $P_r$, by the center vertical displacement, $\delta$, whereas the torsion stiffness of the torsion beam, $S_{\text{eff}}$, is derived by dividing the applied torque, $T_r$, by the torsion angle, $\theta$. They are expressed as follows:

$$K_{\text{eff}} = \frac{P_r}{\delta} \quad \text{and} \quad S_{\text{eff}} = \frac{T_r}{\theta}. \quad \tag{3}$$

Using Eqs. (1)–(3), the effective torsion and bending stiffness can be calculated and obtained (refer to Appendix A).

Fig. 3 demonstrates that the effective torsion and bending stiffness of the torsion beam depend significantly on the geometry of the torsion beam, particularly the height and number of springs. As the height of the spring increases, there is a sharp decrease in the effective torsion stiffness initially, however, this decrease slows down.
2.2. Static actuation characteristics

When a bias voltage, $U$, is applied between the micromirror and one of the electrodes, the electrostatic force and torque, induced by the electrostatic attraction, will rotate the micromirror to an angle of $\theta$ and have a displacement of $\delta$. In the Cartesian coordinate system shown in Fig. 1, the electrode plate and micromirror are characterized by $y = 0$ and $y = h - \delta - x \tan \theta$, respectively. The electrostatic potential, $\Phi$, satisfies the Laplace equation [26]

$$\nabla^2 \Phi = 0. \quad (4)$$

The boundary conditions on the two plates are

$$\Phi_{y=0} \quad \text{and} \quad \Phi|_{y=h-\delta-x \tan \theta} = U. \quad (5)$$

The electrostatic potential, $\Phi$, which satisfies the Laplace Eq. (4), can be expressed as the imaginary part of a complex analytic function, $\varphi$

$$\Phi = \text{Im} \varphi(\zeta), \quad (6)$$

where $\zeta = x + iy$, $i = \sqrt{-1}$, and $(x, y)$ are the Cartesian coordinates in Fig. 1. With the following conformal mapping:

$$\zeta = \left[ -\xi + (h - \delta) / \tan \theta \right] - \pi / \theta. \quad (7)$$

The two plates with an angle of $\theta$ in Fig. 1 are mapped to coplanar plates in the $\zeta$ plane. The corresponding boundary conditions become $\text{Im} \varphi(\zeta) = U$ and $\text{Im} \varphi(\zeta) = U$ on the two plates in the $\zeta$ plane. If the edge effect is neglected, the complex analytic function, $\varphi$, is

$$\varphi(\zeta) = \frac{U}{\pi} \ln \zeta. \quad (8)$$

Substituting Eqs. (7) and (8) into Eq. (6), the electrostatic potential can be written as

$$\Phi = \frac{U}{\theta} \tan^{-1} \frac{y \tan \theta}{h - \delta - x \tan \theta}. \quad (9)$$

The distributed charge density, $\sigma$, on the surface of micromirror is obtained from the electrostatic

 gradually. On the other hand, the effective bending stiffness decreases steadily with the height of the spring, as shown in Fig. 3(a). It should also be noted that the effective torsion and bending stiffness of the torsion beam are highly sensitive to the number of springs. They decrease abruptly, when the number of springs increases, as shown in Fig. 3(b).
potential, \( \Phi \), by [26]

\[
\sigma = -\varepsilon \frac{\partial \Phi}{\partial n} = -\varepsilon \frac{U}{\theta} \frac{\sin \theta}{h - \delta - s \sin \theta},
\]

where \( \varepsilon \) is the permittivity of air (\( \varepsilon = 8.854 \, \text{pF/m} \)); \( n \) is the outward normal on the micromirror pointing to the medium; \( \varepsilon \partial \Phi/\partial n \) stands for the gradient of \( \Phi \) along the normal direction; and \( s \) is the local coordinate along the plate direction with the origin at the center of the micromirror (see Fig. 1). The electrostatic force (per unit surface area) acting on the charge is [26]

\[
f = \frac{1}{2} \frac{\sigma^2}{\varepsilon} = \frac{\varepsilon U^2}{2h^2} \left( \frac{\sin \theta}{h - \delta - s \sin \theta} \right)^2 n = f n, \tag{11}
\]

where \( n \) is the outward unit normal on the micromirror; and \( f \) is the magnitude on \( f \). The force (per unit surface area) in Eq. (11) produces an electrostatic force, \( P_e \), that bends the micromirror down from the initial position, \( \delta = 0 \), and a torque, \( T_e \), that rotates the micromirror away from the flat position, \( \theta = 0 \), simultaneously. Therefore the electrostatic force, \( P_e \), and torque, \( T_e \), are given by

\[
P_e = L \int_{a/2}^{a/2} f \, ds = \frac{\varepsilon U^2 L}{2h^2} \sin \theta \times \left[ \frac{1}{1 - \frac{\delta}{h} - \frac{a_s}{2h} \sin \theta} - \frac{1}{1 - \frac{\delta}{h} - \frac{a_1}{2h} \sin \theta} \right], \tag{12}
\]

and

\[
T_e = L \int_{a/2}^{a/2} s f \, ds = \frac{\varepsilon U^2 L}{2h^2} \times \left[ \frac{1}{1 - \frac{\delta}{h} - \frac{a_s}{2h} \sin \theta} - \frac{1}{1 - \frac{\delta}{h} - \frac{a_1}{2h} \sin \theta} \right. \\
\left. + \ln \frac{1 - \frac{\delta}{h} - \frac{a_s}{2h} \sin \theta}{1 - \frac{\delta}{h} - \frac{a_1}{2h} \sin \theta} \right]. \tag{13}
\]

The gap between the torsion micromirror and electrode plate is smaller compared to the width of the micromirror, i.e., the torsion angle is very small (\( \theta \ll 5^\circ \)), so that \( \tan \theta \approx \sin \theta \approx \theta \). The error in the above solution is on the order of 0.1%. For simplicity, the normalized structure parameters can be adopted as

\[
\alpha = \frac{a_1}{a}, \beta = \frac{a_2}{a}, \Theta = \frac{\theta}{\theta_{cr}} , \Delta = \frac{\delta}{h}, \text{ and } \Delta_s = \frac{h_s}{h}, \tag{14}
\]

where \( \alpha \) and \( \beta \) are two normalized parameters, which define the electrode size and position. These parameters are independent of the width of the torsion micromirror; \( \Theta \) and \( \Delta \) are the normalized torsion angle and displacement of torsion beam; \( \Delta_s \) is the normalized gap distance between the torsion beam and groove; and \( \theta_{cr} = 2h/a \) represents the critical torsion angle. Critical condition of the motion of the torsion micromirror will be restricted by

\[
\Theta + \Delta \leq 1. \tag{15}
\]

Substituting Eq. (14) into Eqs. (12) and (13), the electrostatic force and torque are simplified as

\[
P_e = \frac{\varepsilon U^2 L}{2h\theta_{cr} \Theta} \left[ \frac{1}{1 - \Delta - \beta \Theta} - \frac{1}{1 - \Delta - \alpha \Theta} \right] \tag{16}
\]

and

\[
T_e = \frac{\varepsilon U^2 L}{2\theta_{cr} \Theta^2} \left[ \frac{1 - \Delta}{1 - \Delta - \beta \Theta} - \frac{1 - \Delta}{1 - \Delta - \alpha \Theta} + \ln \frac{1 - \Delta - \beta \Theta}{1 - \Delta - \alpha \Theta} \right]. \tag{17}
\]

In Eqs. (16) and (17), all the structural parameters except \( z \) and \( \beta \), contribute to \( P_e \) and \( T_e \) as coefficients. Since the electrostatic force, \( P_e \), and torque, \( T_e \), are proportional to the micromirror length, \( L \), and square of the applied voltage, \( U \), \( P_e \) and \( T_e \) are very sensitive to the applied voltage.

When the micromirror is driven to bend down and rotate by electrostatic force and torque simultaneously, the vertical displacement and torsion angle of the torsion beam will generate an elastic recovery force, \( P_r = K_{eff} \delta \), and torque, \( T_r = S_{eff} \theta \), which are supported by small vertical and angular displacement assumptions. After the torsion beam contacts with the groove, the elastic recovery force is replaced by \( K'_{eff} (\delta - h_g) + K_{eff} h_g \), and the torque remains constant due to the small angular displacement assumption. The micromirror is balanced through the forces and torques at the static equilibrium condition \([15–18,23]\), i.e., \( P_e = P_r \) and \( T_e = T_r \). The static equations can be derived as follows:

\[
\mathcal{U} = \mathcal{U} - \kappa \left[ \frac{\delta}{1 - \Delta - \beta \Theta} - \frac{\theta}{1 - \Delta - \alpha \Theta} \right] = 0, \quad (0 \leq \Delta \leq \Delta_g), \tag{18a}
\]

and

\[
\mathcal{U} = \mathcal{U} - \kappa' \left[ \frac{\delta^{1/2}}{1 - \Delta - \beta \Theta} - \frac{\theta^{1/2}}{1 - \Delta - \alpha \Theta} \right] = 0, \quad (\Delta_g \leq \Delta \leq 1), \tag{18b}
\]

where \( \kappa_1 = (2S_{eff} \theta_{cr}^2/\varepsilon L)^{1/2} \), \( \kappa_2 = (2K_{eff} h_g^2 / \varepsilon L)^{1/2} \), \( \kappa = \kappa_2 / \kappa_1 \), \( \mathcal{U} = \mathcal{U} / \kappa_1 \), \( \kappa' = (2K'_{eff} h_g^2 \theta_{cr} / \varepsilon L)^{1/2} \), \( \kappa' = \kappa'_2 / \kappa_1 \), \( \eta = \kappa_2 / \kappa'_2 = \kappa / \kappa' \), and \( K'_{eff} = 24Eh_b L / h^3 \). Eqs. (18a) and (18b) are the normalized equations which represent the static relationships between the torsion angle, vertical displacement, applied voltage, electrode parameters, and
all the structural parameters of the torsion beams. Before the torsion beam contacts the groove, the torsion angle, vertical displacement, and applied voltage depend on electrode parameters and all other structural parameters, as given in Eq. (18a). After the torsion beam contacts the groove, the torsion angle, vertical displacement, and applied voltage only depend on electrode parameters and some of the structural parameters of the torsion beams between the micromirror and groove, as given in Eq. (18b). \(k\) and \(k'\) represent the combination of all the structural parameters, which acts as a sensitivity and important design parameter of the torsion and bending effect for the torsion micromirror. They also determine the magnitude of the applied voltage. The normalized voltage is mainly dominated by the normalized electrode parameters (represented by \(a\) and \(b\)) and ratio of the bending and torsion effect, \(\kappa\) (or \(\kappa'\)).

2.3. Pull-in characteristics

Initially, the torsion angle and vertical displacement increase with required applied voltage. However, when the required applied voltage reaches a certain value of \(U_{cr}\), the torsion angle and vertical displacement reach their critical values (\(\Theta_{cr}\) and \(\Delta_{cr}\), respectively). When the required applied voltage exceeds \(U_{cr}\), the torsion beam collapses abruptly to the fixed electrode plane. At this turning point, the torsion angle, vertical displacement, and applied voltage are referred to as pull-in angle, pull-in displacement, and pull-in voltage. This is the well-known pull-in phenomenon.

In order to obtain the pull-in point, an approach derived directly from the static Eq. (18) is used. The pull-in phenomenon obtained by rewriting Eq. (18) is governed by [27]

\[
\psi(\Theta, \Delta) = U + \lambda U',
\]

\(U' = 0,\) \hspace{1cm} (19)

where \(\psi(\Theta, \Delta)\) is a function of the normalized applied voltages \(U\) and \(U'\) for calculating conditional extremum using Langrange method [27]. The function \(\psi(\Theta, \Delta)\) depends on two different normalized parameters, \(\Theta\) and \(\Delta\). \(\lambda\) is the Langrange multiplier.

As a result, the pull-in point satisfies the following equations [27]

\[
\frac{\partial \psi(\Theta, \Delta)}{\partial \Theta} = 0, \quad \frac{\partial \psi(\Theta, \Delta)}{\partial \Delta} = 0, \text{ and } U' = 0.
\]

(20)

Eq. (20) can be further deduced using Eq. (19) as

\[
\frac{\partial U}{\partial \Theta} \frac{\partial U'}{\partial \Delta} - \frac{\partial U}{\partial \Delta} \frac{\partial U'}{\partial \Theta} = 0 \text{ and } U' = 0.
\]

(21)

Eq. (21) can also be deduced by applying the total co-energy method [28] to consider the coupling effect between the torsion and bending. Eq. (21) is a set of nonlinear algebraic equations. The normalized pull-in angle, \(\Theta_{cr}\), and displacement, \(\Delta_{cr}\), are relative to the normalized electrode parameters \(a\) and \(b\), and ratio of the bending and torsion effect, \(\kappa\). They can be numerically solved using MATHEMATICA SOLVER [29], and obtained from Eq. (21). The pull-in voltage, \(U_{cr}\), is derived using Eq. (18), in which the torsion angle and vertical displacement are replaced by the normalized pull-in angle, \(\Theta_{cr}\), and

![Fig. 4. Pull-in parameters versus the ratio of the bending and torsion effect, \(\kappa\), at a certain electrode size and position parameter, \(a = 0.06, b = 0.84\). (a) pull-in angle versus \(\kappa\), (b) pull-in displacement versus \(\kappa\), (c) pull-in voltage versus \(\kappa\).](image-url)
displacement, $\Delta_{cr}$

$$U_{cr} = \kappa_1 \left[ \frac{1 - \Delta_{cr} + \beta \Theta_{cr}}{1 - \Delta_{cr} - \beta \Theta_{cr}} \right]^{1/2}$$  \hspace{1cm} (22)

Eq. (21) indicates clearly how to obtain a specific pull-in angle and displacement through the selection on the electrode size and position, position of the groove, and ratio of the bending and torsion effect. This is particularly useful for the design of torsion micromirrors.

When the groove supporters are not provided in the system and the electrode size and position are given certain values, i.e., $\Delta_s = 1$, $\alpha = 0.06$ and $\beta = 0.84$, the pull-in parameters are sensitive to only one normalized structural parameter, $\kappa$, as demonstrated in Fig. 4. The pull-in angle, which is derived by the coupled model, increases as the ratio of the bending and torsion effect increases. It approaches a value of $\Theta = 0.5236$, which is provided by the torsion model [15–18, 23], as shown in Fig. 4(a). The pull-in displacement, which is given by the coupled model, increases as the ratio of the bending and torsion effect, $\kappa$, decreases. It approaches a value of $\Delta = 1/3$, which is provided by the bending model [30], as shown in Fig. 4(b).

The pull-in voltage, which is computed from the coupled model, increases as the ratio of the bending and torsion effect increases. It approaches a value of $U/\kappa_1 = 0.8365$, which is provided by the torsion model, as shown in Fig. 4(c). Thus, the ratio of the bending and torsion effect is one of the important parameters for the torsion micromirror, which determines the pull-in instability mode of the torsion micromirror.

In order to decrease the applied voltage, it is necessary to reduce the effective torsion coefficient, $\kappa_1$, of the torsion beam. The effective torsion coefficient, $\kappa_1$, and ratio of the bending and torsion effect, $\kappa$, are highly dependent on the effective stiffness of the torsion beam, and can be changed by the geometry of the torsion beam, i.e., the height and number of springs. The effective torsion coefficient decreases drastically as the height or number of the springs increases, as shown in Figs. 5(a) and (b). However, the ratio of the bending and torsion effect lies within the range of low values due to the geometrical change of the torsion beam. According to the former analysis (low value of $\kappa$ will result in the instability due to the bending effect), it is very difficult to apply the torsion beam for applications based on the torsion micromirror. As a result, the U-shaped groove supporters are introduced in the design of the torsion micromirror. The U-shaped groove is necessary to consider in the theoretical model, as it plays a key role in changing the effective bending stiffness of the torsion beam. The U-shaped groove supporters can also act to improve the effective bending stiffness of the torsion beam to a higher value, and therefore the ratio of the bending and torsion effect increases (i.e., $\kappa' > 1$ and $\eta < 1$), so that the instability mode of the torsion micromirror is switched from the instability which is dominated by the bending effect to the instability which is dominated by the torsion effect.

After assigning a certain value, i.e., $\Delta_s = 0.16$, to the normalized gap distance of the groove, pull-in parameters can be derived from the coupled model when the ratio of the bending and torsion effect after contacting with the groove is sufficiently large, i.e., $\kappa' = 10$ and $\eta = 0.01$ (as shown in Fig. 6). When the normalized electrode size and position, $\beta$, exceeds a critical value, the pull-in phenomenon happens. The pull-in angle decreases as the normalized electrode size and position, $\beta$, increases. It has the same trend to that of the torsion model [15–18, 23]. However, the magnitude of the pull-in angle, provided by the coupled model, is smaller than the torsion model, as shown in Fig. 6(a). At the same time, the vertical pull-in displacement is very close to the normalized gap of the groove, $\Delta_s$, as shown in Fig. 6(b). The pull-in voltage obtained from the coupled model is also smaller than that of the torsion model, as shown in Fig. 6(c). Furthermore, the pull-in

![Fig. 5. The effective torsion coefficient, $\kappa_1$, and ratio of the bending and torsion effect, $\kappa$, versus spring parameters of the torsion beam: and (a) $\kappa_1$ and $\kappa$ versus the height of the spring; and (b) $\kappa_1$ and $\kappa$ versus the number of the spring.](image-url)
angle, pull-in displacement, and pull-in voltage have slight differences if \( z \leq 0.2 \). Therefore, it is necessary to consider the position of the groove in the coupled model, otherwise, the theoretical results will be inaccurate. The pull-in phenomenon is mainly determined by the electrode parameter, \( b \), and position of the groove, \( \Delta_g \), when the structure parameter, \( \kappa' \), is significantly large.

3. Numerical and experimental verification

These micromirrors are fabricated using the three-layer-polysilicon surface micromachining process. The SEM micrograph of the torsion micromirror is shown in Fig. 7. The top of the micromirror is supported by two torsion beams, which are particularly designed as spring-shaped and sequentially connected to and supported by the two anchors. There are two electrodes beneath the micromirror, which are connected to the pads to input an applied voltage. The U-shaped grooves are used to maintain the air gap between the micromirror and the electrodes, while the staples are employed to prevent the micromirror from being flushed away during the wet etching processes. In addition, one anchor is connected to another pad to ground the micromirror surface. A group of torsion micromirrors with \( \beta \) between the ranges of 0.3–0.9 is fabricated to verify the relationships between the pull-in angle, pull-in displacement, pull-in voltage, and electrode size and position.

In order to further confirm the theoretical model of the torsion micromirror, coupled-domain FEM/BEM simulations using MEMCAD Cosolve module (a business software ConventorWare\textsuperscript{TM} [31]) are performed. The FEM mechanical solver requires a 3-D mesh while the BEM electrostatic solver requires only a 2-D mesh of the conducting surfaces. The dimensions of the simulated micromirror are listed in Table 1. The simulations are
performed using the relaxation method [31]. The solver iterates the applied voltage so that it is between the ranges of the pull-in voltage. At each of these iterations, an inner relaxation loop is used to confirm or refute convergence for the current applied voltage. In the relaxation loop a BEM solver, which only requires a 2-D mesh of the conducting surfaces, is used to evaluate the electrical forces applied at the surface of the micromirror for a current deformation. These electrical forces are used as the boundary conditions for a mechanical FEM solver to calculate the new deformation. The FEM mechanical solver requires 3-D mesh. The loop continues either (i) until the deformation converges within a given relative criterion (here $10^{-4}$ is used) and then the voltage is below the pull-in voltage, or (ii) the deformation does not converge and the voltage is therefore determined as above the pull-in voltage. The iterations of the applied voltage continue until a given accuracy (here 0.01%) allows sufficient convergence of the pull-in angle. The typical simulation time of each case is about 3 days.

The experimental set-up, in which an optical projection method is used to measure the torsion angle of the micromirror, is illustrated in [23]. The theoretical analysis, numerical simulation, and experimental data which show the variation of the pull-in angle and pull-in voltage due to the change of the normalized electrode size and position are shown in Fig. 8. The theoretical model on the pull-in angle of the torsion micromirror is consistent with the numerical and experimental results with standard deviations of 1% and 4% (as shown in Fig. 8(a)). The prediction of the pull-in voltage from the theoretical model is less than those of the numerical and experimental results within the errors of 3% and 6% (as shown in Fig. 8(b)). It may be due to the spring-shaped torsion beams which work differently.

The rotation wobble induced by the square cross-section of the beams and the stiction between the torsion beams, the staples, and the U-shaped grooves may increase the actual stiffness. Although these only have small effect on the pull-in angle and pull-in displacement, a larger pull-in voltage is required to overcome them.

A typical configuration of the torsion micromirror is analyzed to have comparison between the theoretical data, numerical simulation, and experimental results. This is shown in Fig. 9. The numerical simulation and experimental data follow the prediction of the theoretical model closely and demonstrate significant result. In addition, the deviation of the torsion angle using the theoretical model is within the ranges of 2% and 4%. For the same applied voltage, the simulated and experimental torsion angles are always smaller than the theoretical values. In Fig. 9, the

### Table 1

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<th>Parameters of the electrostatic torsion micromirror</th>
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<tr>
<td>Groove</td>
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<td>Electrode</td>
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Fig. 8. Comparison between theoretical model, numerical simulation, and experimental results of the torsion micromirror. (a) pull-in angle versus $\beta$, (b) pull-in voltage versus $\beta$. 
simulated and measured values of the pull-in voltage, $U_{cr}$, are 1.69 and 1.75 V, respectively. The values of the parameters from the theoretical model are comparable to the values from the experiments and numerical simulations. Table 2 illustrates that the theoretical results are consistent with the simulated and experimental results.

### 4. Conclusions

A theoretical model has been developed to represent the relationships among the applied voltage, torsion angle and vertical displacement. The pull-in effect has been investigated. These include the pull-in angle, pull-in displacement and pull-in voltage, which depend significantly on the normalized electrode size and position, groove position, and ratio of the bending and torsion effect. The pull-in instability mode of the torsion micromirror is mainly determined by the groove position and ratio of the bending and torsion effect. An effective torsion and bending stiffness model is used to optimize the geometry of the torsion beams, and to reduce the pull-in voltage of the torsion micromirror below 2 V for low-voltage applications to ensure that it is compatible with the IC technologies. The pull-in parameters of the torsion micromirror, which are obtained from the theoretical analysis, demonstrate consistency with those of the numerical simulation and experiments.

### Appendix A

The strain energies, $\Pi_1$, $\Pi_2$, $\Pi_3$, $\Pi'_1$, $\Pi'_2$, and $\Pi'_3$ of the torsion micromirror, shown in Fig. 2, are provided as follows:

\[
\Pi_1 = \int_0^{L_1} \frac{(P_0 \zeta - M_0)^2}{2EI_{b1}} \, d\zeta + \int_0^{L_1} \frac{T_0^2}{2GI_{p1}} \, d\xi, \quad (A.1)
\]

\[
\Pi_2 = n \int_0^{H} \frac{(P_0 \zeta - T_0)^2}{2EI_{b2}} \, d\zeta + \sum_{i=1}^{n} \int_0^{H} \frac{[P_0(L_1 + 2(i-1)S + \zeta) - M_0]^2}{2GI_{p2}} \, d\xi
\]

\[
\Pi_3 = \int_0^{L_2} \frac{[-P_0(L_1 + 2nS + \zeta) + M_0]^2}{2EI_{b4}} \, d\xi + \int_0^{L_2} \frac{T_0^2}{2GI_{p4}} \, d\xi, \quad (A.3)
\]

\[
\Pi'_1 = \int_0^{L_1} \frac{[P_0 - P_e \zeta] + P_0L_h - M_0)^2}{2EI_{b4}} \, d\xi + \int_0^{L_2} \frac{T_h^2}{2GI_{p4}} \, d\xi \quad (A.4)
\]

\[
\Pi'_2 = n \int_0^{H} \frac{(P_h \zeta - T_h)^2}{2EI_{b2}} \, d\zeta + \sum_{i=1}^{n} \int_0^{H} \frac{[P_h(L_2 + 2(i-1)S) + P_0L_h - M_0]^2}{2GI_{p2}} \, d\xi
\]
where

\[ A_1 = \frac{L_1^2}{E I_{b1}} + \frac{8nH(L_1 + nS)}{G I_{p2}} + \frac{4nS(L_1 + nS)}{E I_{b3}} + \frac{L_2(2L_1 + L_2 + 4nS)}{E I_{b4}}. \]

and

\[ A_2 = \frac{L_1}{E I_{b1}} + \frac{4nH}{G I_{p2}} + \frac{2nS}{E I_{b3}} + \frac{L_2}{E I_{b4}}. \]

Applying the Castigliano’s second theorem, the torsion angle and vertical displacement of the torsion micromirror are given:

\[ \delta = \frac{\partial II}{\partial P_r} \quad \text{and} \quad \theta = \frac{\partial II}{\partial T_r}. \]

(A.12)

Combined with Eqs. (3) and (A.12), the effective torsion and bending stiffness are provided:

\[ S_{\text{eff}} = \left( \frac{L_1}{2G I_{p1}} + \frac{4nH}{2E I_{b2}} + \frac{2nS}{2G I_{p2}} + \frac{L_2}{2G I_{p4}} \right)^{-1}, \]

(A.13)

\[ K_{\text{eff}} = \frac{Z_a}{Z_b}, \]

(A.14)

where

\[ Z_a = 24\left( \frac{L_1}{E I_{b1}} + \frac{4nH}{G I_{p2}} + \frac{2nS}{E I_{b3}} + \frac{L_2}{E I_{b4}} \right), \]

\[ Z_b = \left( \frac{16n^2H^3}{E I_{b1}} + \frac{24nH^2S}{G I_{p3}} + \frac{8nHS^2}{G I_{p2}} + \frac{64n^3HS^2}{G I_{p2}} + \frac{32n^5S^3}{E I_{b3}} \right) \times \left( \frac{L_1}{E I_{b1}} + \frac{2nS}{E I_{b3}} + \frac{L_2}{E I_{b4}} \right) \]

\[ + \left( \frac{48n^2HS}{G I_{p2}} + \frac{32n^2S^2}{E I_{b3}} \right) \left( \frac{L_1}{E I_{b1}} - \frac{4n^2S^2}{E I_{b3}} + \frac{L_2}{E I_{b4}} \right)^2 \]

\[ + \left( \frac{16H S}{G I_{p2}} + \frac{8nS}{E I_{b3}} \right) \left( \frac{L_1}{E I_{b1}} + \frac{8nS^3}{E I_{b3}} + \frac{L_2}{E I_{b4}} \right) \]

\[ + \frac{32n^2H^2}{G I_{p2}} \left( \frac{2H^2}{E I_{b2}} + \frac{(1 + 2nSt^2)^2}{3HS} + \frac{H^4}{G I_{p3}} \right) \]

\[ + \frac{4L_1L_2}{E^2T_{b1}t_{b4}} \left[ (L_1 + L_2 + 2nS)^2 - L_1L_2 \right. \]

\[ + 2nS(L_1 + L_2) + 8n^2S^2 \].

References


[29] MATHMATICA version 3.0, Wolfram Research, see also in the webpage http://www.wolfram.com

