An approach to the coupling effect between torsion and bending for electrostatic torsional micromirrors

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Abstract

A general theoretical model using the coupling effect between the torsion and bending is presented in this paper, which characterizes the static properties of the electrostatic torsional micromirror, especially its pull-in effect. A set of normalized equations governing the static actuation properties of the torsional micromirror based on the parallel plate capacitor model is derived to demonstrate the relationships between the parameters of static characteristics, such as torsion angle, vertical displacement, and applied voltage. Thereafter, the pull-in effect is investigated specifically to predict pull-in voltage, pull-in angle, and pull-in displacement, which highly depend on the electrode size and position, and ratio of the bending and torsion effect of the torsion beam. The ratio of the bending and torsion effect plays a key role in the pull-in phenomena. It also determines the instability mode of torsional micromirrors dominated by either the torsion or bending effect. Then, a group of torsional micromirrors is fabricated using three-layer-polysilicon micromachining process and measured using an optical projection method to verify the static actuation relation and pull-in effect respectively. The experimental data are processed and analyzed, and the theoretical analysis is in good agreement with the experimental results.

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1. Introduction

Microoptoelectromechanical systems (MOEMS) including optical cross-connects [1,2], optical switches [3], microscanners [4], digital micromirror device (DMD) [5], electrically controlled variable attenuators [6], and etc., are being investigated for the emerging of all-optical telecommunications networks [7], in response to the demand for increased information capacity. With the development of micromachining techniques, electrostatic micromirror as an actuator plays an important role in many of these MOEMS systems.

Various micromirror devices have been reported for various applications in open literatures. Based on their motion types, micromirrors can be simply classified into four categories: deformable micromirror [8], movable micromirror [9], piston micromirror [10], and torsional micromirror [2,11–17]. These four types of micromirrors have been widely applied in recent years with the torsional micromirror being the most interesting among them. The torsional micromirror has been widely used for applications because of its good dynamic response and small possibility of adhesion, for instance in digital projection displays [11,12], spatial light modulators [13,14], optical crossover switches [15], and adaptive optics [16].

The characteristics of the torsional micromirror have been extensively studied [2,15–17–27]. Among these characteristics, the pull-in phenomenon remains a significant phenomenon for electrostatic torsional micromirrors. The electrostatic force/torque overcomes the mechanical force/torque and the movable plate of the micromirror snaps abruptly to the fixed electrode plate, by increasing the applied voltage on the micromirror plates above a certain voltage. The specific pull-in parameters, namely, pull-in voltage, pull-in angle, and pull-in displacement, characterize the performance of the micromirror. The pull-in parameters are determined by the geometrical design of the micromirror and the actuating electrodes. In all applications, a model for the variations of the pull-in parameters with the design
parameters defining the geometry and electrode location of the torsion micromirror is required.

Previous models considered the torsion actuator as one degree-of-freedom (d.f.) actuator. The effect of dimension and location of bottom electrode plate is analyzed [2,15,17–27], and the effect of non-linear strain stiffening of the torsion beam is also considered [20,21]. These cases will be efficient when the vertical displacement of the torsional micromirror due to the electrostatic force applied between the micromirror and electrode plate is very small. However, when the vertical displacement and torsion angle of the torsional micromirror are within values of the same order, the bending and torsion effect are coupled to affect the static characteristics of the torsional micromirror. As a result, the designed pull-in torsion angle using previous models may be insufficiently accurate and is actually far smaller than the real one. Moreover, when the vertical displacement of the torsional micromirror reaches a gap of 10% between the micromirror and electrode plate (approximately 0.5 μm), the phase of reflected light beam is changed by half and even one wavelength of the lights. Thus, it seriously affects the design and usage of relevant optical applications, and may even result in the wrong design, i.e., in spatial light modulators [13,14]. Therefore, accurate analysis for the torsion angle and vertical displacement of the torsional micromirror is very important to optical devices and applications.

In this paper, a similar methodology to the general pull-in model for electrostatic actuators with lumped two coupled degrees-of-freedom [28] is developed for the electrostatic torsional micromirrors. The parallel plate capacitor model, which ignores the fringing effect and residual charges, is used in this theoretical model. The torsion beams and micromirror are assumed to have a flat position initially and to work in the small-deflection (including small vertical and angular displacements) regime. The pull-in characteristics of the torsional micromirror are investigated using the coupled model of the torsion and bending, which dominates the fringing effect and residual charges, is used in this theoretical model. The torsion beams and micromirror are assumed to have a flat position initially and to work in the small-deflection (including small vertical and angular displacements) regime. The pull-in characteristics of the torsional micromirror are investigated using the coupled model of the torsion and bending, which is in full employed normalized forms. The coupled model of the torsion and bending provides a pull-in instability mechanism, which is dominated by the torsion or bending effect. In Section 2, a static mechanical model consisting of a set of normalized equations is derived to represent the relationships between the applied voltage, torsion angle, and vertical displacement of the torsional micromirror. It is also used to predict the relationship between the pull-in voltage, pull-in angle, and pull-in displacement with the geometrical parameters of the micromirror and electrode plate. All the design, fabrication and experiment results of the torsional micromirror are presented in Section 3.

2. Pull-in theoretical model

In the following subsections, the theoretical analysis of an electrostatically actuated torsional micromirror is discussed. The electrostatic force and torque are given using the parallel plate capacitor model. Then, a set of expressions for the pull-in effect, which depends on the structure parameters, is formulated.

2.1. Static actuation characteristics

A schematic diagram of a torsional micromirror and its cross-sectional view are shown in Fig. 1a and b, respectively. The micromirror is supported by two torsion microbeams, which are in turn mounted on two anchors fixed to the substrate. Beneath the micromirror, there are two electrodes on the substrate. The micromirror can be driven to rotate by adding potential between the micromirror and one electrode, and can rotate in the reverse direction if potential is introduced between the micromirror and another electrode instead. Table 1 summarizes all the design variables and definitions required for the analysis of the torsional micromirror.

Four main assumptions of the theoretical model are adopted as: (1) The torsion beams and micromirror with a prismatic cross section, no undercutting, and no overetching are initially flat, parallel, and movable or rotate with respect to the electrodes, which are the fixed ground planes, and the torsion beams have perfect fixed boundary conditions. (2) The torsion beams are operated in the small-deflection regime (including small vertical and angular displacements) until the pull-in occurs, i.e., \( \tan(\theta) \approx \sin(\theta) \approx \theta \), and \( \cos(\theta) \approx 1 \). This will give rise to less than 1% error even at 10°. The small vertical and angular displacements approximations
practically mean the critical torsion angle of the micromirror is far smaller than one ($\theta_{cr} \ll 1$) and the vertical displacement is far smaller than the width of the micromirror ($h/\alpha \ll 1$). At the same time, critical condition of the motion of the torsional micromirror is restricted by $\theta + \Delta \ll 1$. (3) The torsion beams have a negligible residual stress and stress stiffening effect. (4) Any non-uniformity in the electric field due to the curvature is neglected. These assumptions are reasonable for the structure under examination, as they are constructed with initially parallel components and have gaps that are small compared to their lateral dimensions.

When the micromirror is driven to bend down and to rotate by the electrostatic force and torque simultaneously, an elastic recovery force, $K_0$, and torque, $S_0\theta$, are generated from the vertical displacement and torsion angle of the torsion beams using small vertical and angular displacement assumptions, respectively. The micromirror is balanced through the forces and torques at the static equilibrium condition [18–21,26,28], and the static equations are derived

$$K_0\theta = 2.2Eh_l/t$$
$$S_0\theta = 2.2Eh_l/t$$

(2) with inertias $h_l$ and $I_p$ of the rectangular cross-section being [28]

$$h_l = \frac{1}{12} w^2 t$$
$$I_p = \frac{1}{3} w^3 t$$

(3)

The model can also be used in other kinds of torsional micromirrors with the different geometries of their cross-section. Eq. (1) can be reconstructed using the normalized parameters, and is rewritten as

$$\bar{V} = \left[\frac{1}{1} - \frac{1}{1 - a} \right] / \left[1 - \frac{1}{1 - a} \right]$$
$$\bar{V} = V - \bar{V} $$

(4)

where $k_1 = 2\sqrt{2/b_0 L/2}$, $k_2 = 2\sqrt{2/b_0 L/2}$, $\kappa = 2\bar{V}/L$, and $V = V/s_0$. Eq. (4) is the normalized equation representing a static relationship between the torsion angle, vertical displacement, applied voltage, and all the structural parameters of the torsional micromirror. $\kappa$ represents the combination of all the structural parameters (except for the normalized electrode parameters) of the torsional micromirror, and only acts as a sensitivity coefficient of the torsion and bending effect for the torsional micromirror. Therefore, the normalized voltage is mainly determined by the normalized electrode parameters (represented by $a$ and $\beta$) and ratio of the bending and torsion effect, $\kappa$. From this point of view, one can conclude that the electrode size and position parameters, $a$ and $\beta$, and ratio of the bending and torsion effect, $\kappa$, are important variables and parameters in determining the static behavior of the torsional micromirror.
torque angle and vertical displacement reach their critical values, \( \theta_{cr} \) and \( \Delta_{cr} \). When the required applied voltage exceeds \( V_{cr} \), the torsional micromirror collapses abruptly to the fixed electrode plate. At the turning point, the torsion angle, vertical displacement, and applied voltage at this stage are referred as pull-in angle, pull-in displacement, and pull-in voltage respectively. This is the well-known pull-in phenomenon.

Since the critical pull-in angle, \( \theta_{cr} \), and pull-in displacement, \( \Delta_{cr} \), determine the angular range over which the micromirror can be smoothly driven, they are important parameters in the applications where the micromirror operates in continuous angles, for instance in a laser scanner. Besides, the critical pull-in voltage \( V_{cr} \) is another important parameter to be considered in practical usage. To be compatible with IC components, the torsional micromirror is normally operated at 5 V, and to prevent electrical breakdown, the pull-in voltage should generally be as low as possible.

In order to obtain the pull-in point, an approach derived directly from the static Eq. (4) is used. The pull-in phenomenon by rewriting Eq. (4) is governed by

\[
\begin{align*}
\frac{\partial f(\theta, \Delta)}{\partial \theta} &= V + \lambda V', \\
\frac{\partial f(\theta, \Delta)}{\partial \Delta} &= 0,
\end{align*}
\]

where \( f(\theta, \Delta) \) is a function which depends on two independent normalized parameters, \( \theta \) and \( \Delta \), and \( \lambda \) is the Langrange multiplier. Therefore, the pull-in point should satisfy the following equations:

\[
\begin{align*}
\frac{\partial f(\theta, \Delta)}{\partial \theta} &= V + \lambda V' = 0, \\
\frac{\partial f(\theta, \Delta)}{\partial \Delta} &= 0, \quad V' = 0
\end{align*}
\]

Eq. (6) can be further deduced using Eq. (5) as

\[
\begin{align*}
\frac{\partial^2 V}{\partial \theta^2} - \beta \frac{\partial V}{\partial \theta} = 0,
\end{align*}
\]

Eq. (7) is also deduced from applying the total co-energy method [28] for considering the coupling effect between the torsion and bending in the torsional micromirror. Eq. (7) is a set of nonlinear algebraic equations. The pull-in angle, \( \theta_{cr} \), and pull-in displacement, \( \Delta_{cr} \), are relative to the electrode parameters \( \alpha \) and \( \beta \), and ratio of the bending and torsion effect, \( \kappa \). They can be numerically solved using MATHEMATICA SOLVER [31], and obtained from Eq. (7). The pull-in voltage, \( V_{cr} \), is derived using Eq. (4), in which the torsion angle and vertical displacement are replaced with the pull-in angle, \( \theta_{cr} \), and pull-in displacement, \( \Delta_{cr} \), respectively.

\[
V_{cr} = \kappa \left[ \frac{\theta_{cr}^2}{(1 - \Delta_{cr})/(1 - \Delta_{cr} - \beta \theta_{cr})} - \frac{1}{\alpha \theta_{cr} (1 - \Delta_{cr} - \alpha \theta_{cr})} + \ln(1 - \Delta_{cr}/(1 - \Delta_{cr} - \alpha \theta_{cr}))/((1 - \Delta_{cr} - \alpha \theta_{cr})) \right]^{1/2}
\]

Eq. (7) indicates clearly how to obtain a specific pull-in angle, pull-in displacement through the selection of the electrode size and position, and ratio of the bending and torsion effect. Figs. 2-4 demonstrate that the normalized pull-in angle, pull-in displacement, and pull-in voltage are strongly dependent on the structure parameters, including the electrode size and position and ratio of the bending and torsion effect.

The torsional micromirror without pull-in can also be derived from the coupled model of the bending and torsion below the critical values when the ratio of the bending and torsion effect is sufficiently large, i.e., \( \kappa = 10 \), as shown in Fig. 2a. It is similar to that of the torsion model [18-21, 26] (pull-in condition is \( \beta \theta_{cr} = 0.4404 \), at \( \alpha = 0 \), i.e., if \( \beta \leq 0.4404/\theta_{cr} = 1 \), no pull-in happens. The pull-in angle, provided by the coupled model of the bending and torsion is very close to the experimental results of Bühler et al. [32] and Toshiyoshi and Fujita [2]. At the same time, the vertical pull-in displacement is very small, as shown in Fig. 2b. The pull-in voltage obtained from the coupled model of the bending and torsion is also in good agreement with the torsion model, as shown in Fig. 2c. When the ratio of the bending and torsion effect is sufficiently large, the effective bending stiffness is far larger than the effective torsion stiffness, so that the vertical displacement is very small, and the torsion effect dominates. Therefore, the coupled model of the bending and torsion can be simplified to the torsion model. At the same time, the pull-in angle, pull-in displacement, and pull-in voltage have slightly differences if \( \alpha \leq 0.2 \), i.e., the pull-in phenomenon is mainly determined by the electrode parameter, \( \beta \), if the structure parameter, \( \kappa \), is significantly large.

When the bending effect is of the same order to the torsion effect, i.e., \( \kappa = 3 \), the pull-in phenomenon of the torsional micromirror is very different from that of the torsion model, as shown in Fig. 3. The pull-in angle, pull-in displacement, and pull-in voltage strongly depend on the electrode size and position. The pull-in angle, provided by the coupled model of the bending and torsion, is lower than that of the torsion model. For small parameter, \( \beta \), the pull-in angle is shown to be sensitive to the parameter, \( \alpha \). Moreover, pull-in phenomenon can take place at every electrode parameter, as shown in Fig. 3a. At the same time, the vertical pull-in displacement is not small, and has the same order compared with the pull-in angle, especially for small parameter, \( \beta \). The pull-in phenomenon is dominated by the bending model [33] as shown in Fig. 3b. The pull-in voltage obtained from the coupled model of the bending and torsion is very different from that of the torsion model, as shown in Fig. 3c. In this situation, the effective bending stiffness is comparable to the effective torsion stiffness. The torsion model cannot be used to replace the coupled model of the bending and torsion. Otherwise, a bigger error will be involved.

When the electrode size and position are given certain values, i.e., \( \alpha = 0.06 \) and \( \beta = 0.84 \), the pull-in parameters are sensitive to the only normalized structural parameter, \( \kappa \), as demonstrated in Fig. 4. The pull-in angle, which is given
by the coupled model of the bending and torsion, increases as the ratio of the bending and torsion effect, \( \kappa \), increases. It approaches a certain value of \( \Theta = 0.5236 \), which is provided by the torsion model, as shown in Fig. 4a. The pull-in displacement, which is given by the coupled model of the bending and torsion, increases as the ratio of the bending and torsion effect, \( \kappa \), decreases. It approaches a certain value of \( \Delta = 1/3 \), which is provided by the bending model [33], as shown in Fig. 4b. The pull-in voltage, which is computed from the coupled model of the bending and torsion, increases as the ratio of the bending and torsion effect, \( \kappa \), increases. It approaches a certain value of \( V/\kappa = 0.8365 \), which is provided by the torsion model, as shown in Fig. 4c. Thus, besides the parameters given by the electrode size and position, the ratio of the bending and torsion effect, \( \kappa \), is another important parameter to the torsional micromirror, which dominates the pull-in instability mode of the torsional micromirror.

3. Experimental verification

3.1. Parameters of the micromirror

Based on the previous analysis, we have designed a group of single torsional micromirrors, whose parameters are listed in Table 2, in order to verify the static actuation relationship and pull-in effect. These micromirrors are fabricated using the three-layer-polysilicon surface micromachining method. This process uses an isolation layer of nitride (0.6 μm thick) on the silicon wafer substrate, three structural layers of polysilicon (0.5 μm, 2 μm and 1.5 μm thick respectively), and two sacrificial layers of phosphosilicate glass (2 μm and 0.75 μm thick respectively) sandwiched between the three polysilicon layers. Finally, on top of the last polysilicon layer (Poly 2) a metal layer (0.5 μm thick) is deposited as the reflection surface. Forty-nine percent hydrofluoric acid (HF) is used to release the structure.
The SEM graph of the single torsional micromirror is shown in Fig. 5. At the top, the micromirror is supported by two torsion beams, which are sequentially connected to and supported by two anchors. Beneath the micromirror, two electrodes are connected to the pads to input an applied voltage. In addition, one anchor is connected to another pad to ground the micromirror surface. In this design, the micromirror is divided into two parts connected by a $6 \mu m \times 6 \mu m$ bridge to increase the length of the torsional micromirror.

### 3.2 Experimental verification

The experimental set-up, in which an optical projection method is used to measure the torsion angle of the micromirror, is illustrated in [26]. The comparison of the experimental results and theoretical data is shown in Fig. 6. The curves...
Fig. 4. Pull-in angle, pull-in displacement, and pull-in voltage vs. the ratio of the bending and torsion effect, $\kappa$, at a certain electrode size and position parameter, $\alpha = 0.06$, $\beta = 0.84$. (a) pull-in angle vs. $\kappa$, (b) pull-in displacement vs. $\kappa$, (c) pull-in voltage vs. $\kappa$.

Fig. 5. The SEM graph of the torsional micromirror.

Fig. 6. Comparison between the experimental results and theoretical models for the torsional micromirror.
torsion beams become small and rounded by over-etching in

It may be due to the fact that the square cross-section of

and torsion. For the same applied voltage, the experimental

theoretical solution using the coupled model of the bending

torsion angle is within the range of 2%, when compare to the

theoretical analysis closely. However, the deviation of the

experiment of the micromirror is also shown in Fig. 6.

The variation range of the experimental data at every exper-

ments (actually these lines do not exist). The star-shaped

the dash lines just denote the theoretically possible defor-

mations of the torsional micromirror, and

the actual deformations of the torsional micromirror, and

angle using the torsion model. The solid lines represent for

the width of the torsion beam is 1.55

4. Conclusions

A set of normalized equations has been derived to repre-

sent the relationships between the applied voltage, torsion

angle and vertical displacement, using the parallel-plate

 capacitor model. The pull-in effect has been investigated.

These include the pull-in angle, pull-in displacement and

pull-in voltage, which depend significantly on the normal-

ized electrode size and position, and ratio of the bending

and torsion effect. The mode of the pull-in instability of

a torsional micromirror is mainly determined by the ratio

of the bending and torsion effect. A group of torsional

micromirrors has been designed and fabricated using the

three-layer-polysilicon surface micromachining process,

and which is also used to verify the theoretical model.

The pull-in angle and pull-in voltage of the torsional mi-

cromirror, which are obtained from the theoretical analysis,

reconile with the experimental results within the errors of

1% and 2%, respectively.

Table 3

Comparison of experimental results and theoretical models at the pull-in point, where $a = 3.24 H$

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>$\theta_{cr}$ (°)</th>
<th>$V_{cr}$ (V)</th>
<th>$\Delta V$ (V)</th>
<th>$\Delta \theta$ (°)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>0.4198</td>
<td>17.4</td>
<td>0.0778</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>Coupled model</td>
<td>0.4208</td>
<td>17.7</td>
<td>1.7</td>
<td>0.0786</td>
<td>1.0</td>
</tr>
<tr>
<td>Torsion model</td>
<td>0.5236</td>
<td>24.7</td>
<td>20.1</td>
<td>15.5</td>
<td>100%</td>
</tr>
</tbody>
</table>

Notes: Error $1 \equiv \left| \frac{\theta_{cr} - \theta_{exp}}{\theta_{cr}} \right| \times 100%$, Error $2 \equiv \left| \frac{V_{cr} - V_{exp}}{V_{cr}} \right| \times 100%$, and Error $3 \equiv \left| \frac{\Delta \theta - \Delta \theta_{exp}}{\Delta \theta_{exp}} \right| \times 100%$.

References


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Biographies

Jian-Ming Huang received his BEng in Engineering Mechanics, BS (mech) in Applied Mathematics, MEng and PhD degrees in Solid Mechanics, all from Tsinghua University, PR China in 1994. He graduated in 1998 and 1999, respectively. He began to concentrate on MEMS technology in July 1999 when he worked as a Research Fellow in Institute of High Performance Computing, Singapore. In July 2001, he joined School of Electrical & Electronic Engineering, Nanyang Technological University, Singapore. He is editor of Optics and Lasers Engineering and was associate editor with Experimental Mechanics. He is a chartered engineer, fellow of SPIE and member of IMechE and OSA. He is also R&D chair of the Photonics Association (Singapore) and Chair of SPIE Singapore Chapter.

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References


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