ABSTRACT
Microparticle trapping and manipulation using the optical force in double coupled ring resonator (DCRR) through wavelength tuning is reported in this paper. The DCRR consists of two coupled identical rings, and a waveguide that is side-coupled to the two rings. The microparticles are propelled by optical radiation force and trapped by optical gradient force, which are both generated by the evanescent field around the ring resonators. The different directions of relevant radiation forces on a polystyrene particle with diameter of 0.5 µm are analyzed at wavelengths of 1.567 µm, 1.569 µm, 1.572 µm, and 1.577 µm, which correspond to different particle moving directions. This versatile system has lots of potential applications in biological or chemical areas, such as micro-drug delivery, single cell manipulation, or micro-reactant transportation.

KEYWORDS
Microparticle trapping, double coupled ring resonator (DCRR), optical force

INTRODUCTION
Evanescent field around a sub-micro-optical waveguide can produce an optical trapping force on micro-particles or cells, that’s verified by several experiments [1-4]. The trapping principle is based on the optical gradient force caused by intensity gradient, which is same as optical tweezers. However, in traditional optical tweezers, the radiation force is canceled by the gradient force on propagation direction using a well focused beam to cause intensity gradient on this direction [5]. For a waveguide, it’s difficult to generate intensity gradient on the propagation direction. The alternative way is to use an opposite propagated beam to cancel the optical radiation force, which will complicate the setup. On the other hand, the optical ring resonator is used to enhance the optical force by intensity enhancement, which is applied in particle transportation and circulation [6-7]. It is more useful if the direction of radiation force is controllable.

DESIGN AND THEORETICAL ANALYSIS
To achieve this target, a DCRR is introduced in this paper, which consists of two coupled identical rings, and a waveguide that is side-coupled to the two rings as shown in Fig. 1(a). The microparticles on the waveguide are propelled to the ring by gradient force. Fig. 1(b) illustrates the optical forces caused by the evanescent field around the waveguide/ring, which include the gradient force ($F_g$) and the radiation force ($Fs$) [1]. The trapped micro-particles are continuously propelled by the radiation force and cannot be stopped either in waveguide case or single ring case. To stop the microparticles, an opposite radiation force is introduced by introducing a converse propagating light through multi-couplers in DCRR [8]. When there are two lights propagating in opposite directions, the radiation forces are also in opposite direction. The resultant of the two forces determine the moving directions of the particle as shown in Fig. 1(c–e).

The radiation force is proportional to the intensity in the waveguide/ring. In the DCRR, the intensities of clockwise light and anticlockwise light are related to the coupling coefficients between the rings and the waveguide. The optical fields in each segments of the DCRR are related by transfer matrix (TM), which can be expressed as [8-9]
DCRR is illustrated by Fig. 2(b) for easy understanding shown in Fig. 2(a). The equivalent optical path of the right/left propagation directions in the waveguide as where optical paths and phase delays from the input to (c – i) equivalent optical path of the DCRR. The possible (k)

\[
\begin{align*}
X_\alpha^{a/c} &= \left( \begin{array}{c} r_\alpha \\ \mp j \alpha t_\alpha \\ r_\alpha \\ \mp j \alpha t_\alpha \\ r_\alpha \\ \mp j \alpha t_\alpha \\ \end{array} \right) \left( \begin{array}{c} X_\alpha^{a/c} \\ X_\alpha^{a/c} \\ X_\alpha^{a/c} \\ X_\alpha^{a/c} \\ X_\alpha^{a/c} \\ X_\alpha^{a/c} \\ \end{array} \right), \quad X = A, B
\end{align*}
\]

introduces a π/2 phase delay to the propagating light, which leads to the presence of the imaginary unit in Eq. (1) and (2). There are totally ten possible optical paths and four possible coupling-caused phase delays (0 = 2π, π = 3π, 0.5π = 2.5π, 1.5π) as shown in Fig. 2(c-i). Two possible phase delays are for transmission (0 & π) and two for reflection (0.5π & 1.5π). Considering the propagation-caused phase delays on the ring \( \phi = \beta L \) and on the waveguide \( \varphi_n = \beta L_{rn} \), the relationships of optical fields between the couplers can be expressed as

\[
\begin{align*}
C_\alpha^{a/c} &= \left( \begin{array}{c} A_\alpha^{a/c} \\ B_\alpha^{a/c} \end{array} \right), \quad \varphi_n = \beta L_{rn},
\end{align*}
\]

"Figure 2. (a) The optical path in the DCRR; and (b) the equivalent optical path of the DCRR. The possible optical paths and phase delays from the input to (c – i) transmission; and (j – l) reflection."

\[
\begin{align*}
&\begin{cases}
X_1^{a/c} = \left( \begin{array}{c} r_1 \\ \mp j \alpha t_1 \\ r_1 \\ \mp j \alpha t_1 \\ r_1 \\ \mp j \alpha t_1 \\ \end{array} \right) \left( \begin{array}{c} X_1^{a/c} \\ X_1^{a/c} \\ X_1^{a/c} \\ X_1^{a/c} \\ X_1^{a/c} \\ X_1^{a/c} \\ \end{array} \right), \\
C_1^{a/c} = \left( \begin{array}{c} r_c \\ \mp j \alpha t_c \\ r_c \\ \mp j \alpha t_c \\ r_c \\ \mp j \alpha t_c \\ \end{array} \right) \left( \begin{array}{c} C_1^{a/c} \\ C_1^{a/c} \\ C_1^{a/c} \\ C_1^{a/c} \\ C_1^{a/c} \\ C_1^{a/c} \\ \end{array} \right),
\end{cases}
\end{align*}
\]

where \( r_\alpha \) and \( t_\alpha \) (\( X = A, B, C \)) are reflectivity and transmissivity of the couplers, \( j \) is the imaginary unit (i. e. \( j^2 = -1 \)). The superscript ‘a/c’ indicates the anticlockwise/clockwise directions in the rings or right/left propagation directions in the waveguide as shown in Fig. 2(a). The equivalent optical path of the DCRR is illustrated by Fig. 2(b) for easy understanding of the light propagating processes. A single ring-ring light coupling or ring-waveguide light coupling introduces a π/2 phase delay to the propagating light, which leads to the presence of the imaginary unit in Eq. (1) and (2). There are totally ten possible optical paths and four possible coupling-caused phase delays (0 = 2π, π = 3π, 0.5π = 2.5π, 1.5π) as shown in Fig. 2(c-i). Two possible phase delays are for transmission (0 & π) and two for reflection (0.5π & 1.5π). Considering the propagation-caused phase delays on the ring \( \phi = \beta L \) and on the waveguide \( \varphi_n = \beta L_{rn} \), the relationships of optical fields between the couplers can be expressed as

\[
\begin{align*}
&\begin{cases}
X_\alpha^{a/c} = \left( \begin{array}{c} A_\alpha^{a/c} \\ B_\alpha^{a/c} \end{array} \right), \\
C_\alpha^{a/c} = \left( \begin{array}{c} A_\alpha^{a/c} \\ B_\alpha^{a/c} \end{array} \right),
\end{cases}
\end{align*}
\]

a and \( a_n \) are the field attenuation factor on the ring and the waveguide, respectively. \( L \) and \( L_{rn} \) are the circumference of the ring and the length of the waveguide between the two rings, respectively. \( \beta = 2nN_{eff}/\lambda \) is the propagation constant, where \( N_{eff} \) is the effective refractive index and \( \lambda \) is the wavelength.

For a single ring resonator with one bus waveguide, a wavelength \( \lambda_n \) corresponding to the \( n \)-order Whispering Gallery mode means that the corresponding propagation-caused phase delay, \( \varphi_n \), is equal to \( 2n\pi \). The other hand, the coupling-caused phase delay of the single resonator for arbitrary wavelengths, \( \varphi_n \), is always equal to \( \pi \). This results a π phase difference (i. e. \( \Delta \varphi = \varphi_n - \varphi_c = \pi \)) between the light beam, which directly passes through the bus waveguide and the light beam, which is coupled out from the ring. Thus, the two beams cancel each other and produce a minimum transmission at this wavelength and an absorption peak around it. For the DCRR, if the two rings are same size as the single ring, the propagation-caused phase delay in each ring, \( \varphi \), is still equal to \( 2n\pi \) at this wavelength. However, the phase difference \( \Delta \varphi \) between the light beam, which directly passes through the bus waveguide and the light beam, which is coupled out from the two rings is not equal to \( \pi \). Instead, it is equal to the resultant of the two coupling-caused phase delays for transmission (\( \Delta \varphi = \varphi_n - \varphi_n \approx 2n\pi \)) at this wavelength. In order to get the minimum value of transmission, the phase difference \( \Delta \varphi \) must be equal to \( \pi \). Thus, the only approach is to change the propagation-caused phase delay \( \beta L \) by shifting the wavelength \( (\lambda = \lambda_n \pm \Delta \lambda) \). It means the absorption peak of DCRR is splitted into several peaks comparing to the absorption peak of the single ring resonator. Assuming the amplitude of the input as 1 (i. e. \( A_r = 1 \)), and the amplitude of the
reflection from output port as zero (i.e. \( B_z = 0 \)), the 22 amplitudes can be totally obtained by solving Eqs. (1–7) once the design is confirmed.

The optical force calculation on the 0.5-µm polystyrene sphere is based on the time-independent Maxwell stress tensor as \[ F_T = \int \left( \bar{T} \cdot \hat{m} \right) dS \] where \( \bar{T} \) is the electric displacement, \( \bar{E} \) is the electric field, \( \bar{B} \) is the magnetic flux field, \( H \) is the magnetic field, and \( \hat{m} \) is the isotropic tensor. The total optical force \( F_T \) acting on the polystyrene particle can be obtained by integrating the time-independent Maxwell stress tensor on the particle surface, and is given by \[ F_T = \int \left( \bar{T} \cdot \hat{m} \right) dS \]

where \( \hat{m} \) is the surface normal vector.

RESULTS AND DISCUSSIONS

Based on the finite difference time domain (FDTD) method and TM method, the wavelength response of transmittivity and circulating powers are simulated. In the simulations, the two rings have the same radius \( R \) of 2.5 µm, width \( W \) of 0.45 µm, ring-ring gap \( G_{rr} \) of 0.1 µm, waveguide - ring gap \( G_{wr} \) of 0.1 µm, thickness \( H \) of 0.4 µm, and refractive index \( N \) of 2 (Si₃N₄). In FDTD simulation, only the TE mode is used as the light source. In TM simulation, the coupling coefficients are calculated based on the coupling theory [11].

The simulated wavelength response of the transmittivity by FDTD method and TM method are in good agreement with each other as shown in Fig. 3(a).

The wavelength response of the circulating power in the segment between the ports C₄ and B₄ is simulated by TM method, as shown in Fig. 3(b). The power propagating along the clockwise direction \( P_c \) is shown as the black solid line and along the anticlockwise direction \( P_a \) is shown as the red dash line. When the wavelength is 1.567 µm, \( P_a \) is higher than \( P_c \). Therefore, the resultant radiation force is along the clockwise direction and the sphere is propelled to perform a clockwise circulation due to the fact that \( P_a \) is lower than \( P_c \). When the wavelength is in the range from 1.575 µm to 1.581 µm, \( P_a \) is close to \( P_c \), which means the resultant radiation force is very small and the movement of the sphere is also quite slowly. For applications which doesn’t require exact particle stopping, it is more practical to choose this wavelength range than the wavelength of 1.569 µm. This is because the former enables a larger wavelength tolerance and has a higher trapping (gradient) force (due to higher circulating power) than the later.

The wavelength response of the circulating power is symmetric to the wavelength 1.5775 µm, thus four representative wavelengths (1.567 µm, 1.569 µm, 1.572 µm, 1.577 µm) can be used to analyze the values of \( P_c \) and \( P_a \) at all segments. The relations are listed in Table 1. The attractive segment is between A₄ and C₄, on which the circulating power \( P_c \) is always not larger than \( P_a \) at any wavelengths. When the wavelength is at 1.567

![Figure 3. The wavelength response curves of (a) the transmittivity and (b) the clockwise circulating power (black solid line) and the anticlockwise circulating power (red dash line) in the segment between the port C₄ and B₄.](image-url)
µm, the spheres are anticlockwise circulating on ring 1 and ring 2. When the wavelength is changed to 1.572 µm, the spheres on ring 2 are clockwise circulating, the spheres on segment C₁ – A₂ of ring 1 are clockwise circulating, and on segment A₄ – C₁ are anticlockwise circulating. This means that the spheres are gathered at coupling C region and transported to the ring 2. Thus, the ring 2 could be used as a microsphere collector.

The optical force on the sphere is calculated by using finite element method (FEM) based on Maxwell stress tensor method. The numerical results show that the optical force on the polystyrene sphere (diameter of 0.5 µm) is 1.3 pN/mW, and the radiation force (horizontal force) is 0.21 pN/mW when the wavelength is 1.572 µm and the position is on the segment C₄ – B₄.

Table 1. The relations between the clockwise power and the anticlockwise power in the all segments.

<table>
<thead>
<tr>
<th>Wavelength (µm)</th>
<th>1.567</th>
<th>1.569</th>
<th>1.572</th>
<th>1.577</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₃ – A₂</td>
<td>Pₐ &gt; Pₖ</td>
<td>Pₐ = Pₖ</td>
<td>Pₐ &lt; Pₖ</td>
<td>Pₐ ≈ Pₖ</td>
</tr>
<tr>
<td>C₄ – B₄</td>
<td>Pₐ &gt; Pₖ</td>
<td>Pₐ = Pₖ</td>
<td>Pₐ &lt; Pₖ</td>
<td>Pₐ ≈ Pₖ</td>
</tr>
<tr>
<td>A₄ – C₁</td>
<td>Pₐ &gt; Pₖ</td>
<td>Pₐ = Pₖ</td>
<td>Pₐ &lt; Pₖ</td>
<td>Pₐ ≈ Pₖ</td>
</tr>
<tr>
<td>C₂ – B₂</td>
<td>Pₐ &gt; Pₖ</td>
<td>Pₐ = Pₖ</td>
<td>Pₐ &lt; Pₖ</td>
<td>Pₐ ≈ Pₖ</td>
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<tr>
<td>A₃ – B₃</td>
<td>Pₐ &gt; Pₖ</td>
<td>Pₐ = Pₖ</td>
<td>Pₐ &lt; Pₖ</td>
<td>Pₐ ≈ Pₖ</td>
</tr>
</tbody>
</table>

CONCLUSIONS

The DCRR for microparticles trapping and manipulation is theoretical analyzed in this paper. The different directions of relevant radiation forces on a polystyrene particle with diameter of 0.5 µm are analyzed at wavelengths of 1.567 µm, 1.569 µm, 1.572 µm, and 1.577 µm, which correspond to different particle moving directions. The total optical force on polystyrene sphere is 1.3 pN/mW, and the radiation force is 0.21 pN/mW when the wavelength is 1.572 µm and the position is on the segment C₄ – B₄. The versatile system has lot of potential applications in biological or chemical areas, such as micro-drug delivery, single cell manipulation, or micro-reactant transportation.

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